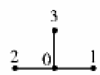
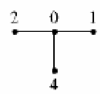
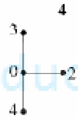
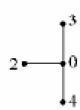
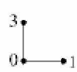
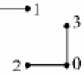
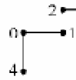
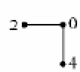
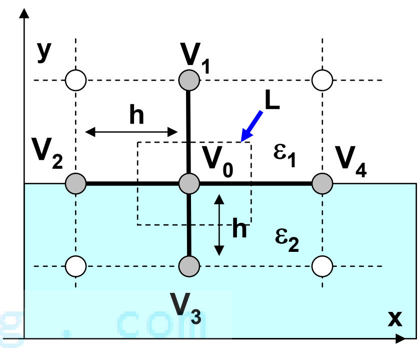
❖ Công thức tổng quát:

Finite Difference Approximations at Boundary Points.

Description	Figure	Cartesian Equation	Cylindrical Equation
1. Bottom edge		$4V_0 = V_1 + V_2 + 2V_3$	$4V_0 = V_1 + V_2 + 4V_3$
2. Top edge		$4V_0 = V_1 + V_2 + 2V_4$	$4V_0 = V_1 + V_2 + 2V_3$
3. Left edge		$4V_0 = 2V_1 + V_3 + V_4$	$8V_0 = 4V_1 + (\frac{2i+1}{i})V_3 + (\frac{2i-1}{i})V_4$
4. Right edge		$4V_0 = 2V_1 + V_3 + V_4$	$8V_0 = 4V_2 + (\frac{2i+1}{i})V_3 + (\frac{2i-1}{i})V_4$
5. Bottom left corner point		$2V_0 = V_1 + V_3$	$3V_0 = V_1 + 2V_3$
6. Bottom right corner point		$2V_0 = V_2 + V_3$	$3V_0 = V_2 + 2V_3$
7. Top left corner point		$2V_0 = V_1 + V_4$	$3V_0 = V_1 + 2V_4$
8. Top right corner point		$2V_0 = V_2 + V_4$	$3V_0 = V_2 + 2V_4$

❖ Biên hai môi trường lý tưởng:

- Trên biên ta có: $\partial V/\partial n = 0$.
- Áp dụng luật Gauss: $\oint_L \varepsilon E_n d\ell = 0$



$$\Rightarrow \oint_L \varepsilon \frac{\partial V}{\partial n} d\ell = 0$$

$$\varepsilon_1 \frac{V_1 - V_0}{h} h + \varepsilon_1 \frac{V_2 - V_0}{h} \frac{h}{2} + \varepsilon_2 \frac{V_2 - V_0}{h} \frac{h}{2} + \varepsilon_2 \frac{V_3 - V_0}{h} h + \varepsilon_2 \frac{V_4 - V_0}{h} \frac{h}{2} + \varepsilon_1 \frac{V_4 - V_0}{h} \frac{h}{2} = 0$$

$$\Rightarrow V_0 = V_{i,j} = \frac{1}{4}\left(\frac{2\varepsilon_1}{\varepsilon_1 + \varepsilon_2} V_1 + \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} V_3 + V_2 + V_4 \right)$$

❖ Biên hai môi trường bất kỳ:

- Tổng quát ta tính các hệ số theo ε_r:

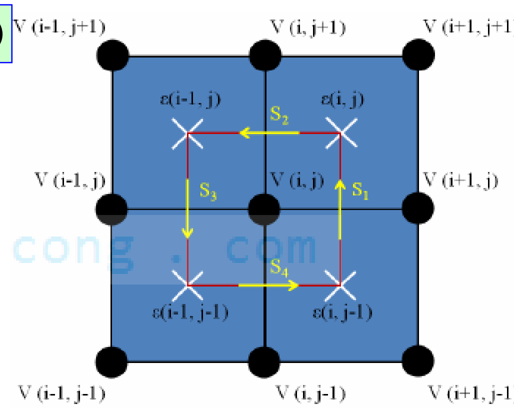
$$a_0 = \varepsilon(i,j) + \varepsilon(i-1,j) + \varepsilon(i,j-1) + \varepsilon(i-1,j-1)$$

$$a_1 = \tfrac{1}{2}\left[\varepsilon(i,j) + \varepsilon(i,j-1)\right]$$

$$a_2 = \tfrac{1}{2}\left[\varepsilon(i,j) + \varepsilon(i-1,j)\right]$$

$$a_3 = \tfrac{1}{2}\left[\varepsilon(i-1,j-1) + \varepsilon(i-1,j)\right]$$

$$a_4 = \tfrac{1}{2}\left[\varepsilon(i-1,j-1) + \varepsilon(i,j-1)\right]$$



- Công thức tính thế tại i, j:

$$V_{i,j} = \frac{1}{a_0}\left[a_1 V_{i+1} + a_2 V_{j+1} + a_3 V_{i-1} + a_4 V_{j-1} + \frac{\rho h^2}{\varepsilon_0}\right]$$

(Công thức của James R. Nagel)