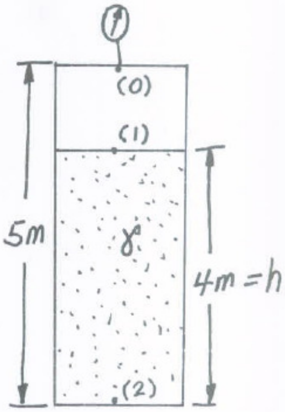
2.2 A closed, 5-m-tall tank is filled with water to a depth of 4 m. The top portion of the tank is filled with air which, as indicated by a pressure gage at the top of the tank, is at a pressure of 20 kPa. Determine the pressure that the water exerts on the bottom of the tank.



$$p_0 = 20 \times 10^3 \frac{N}{m^2} = p_1$$

$$p_2 = p_1 + \gamma h = 20 \times 10^3 \frac{N}{m^2} + 9.80 \times 10^3 \frac{N}{m^3}(4m)$$

$$= 59.2 \times 10^3 \frac{N}{m^2} = \underline{\underline{59.2\ kPa}}$$

2.3 A closed tank is partially filled with glycerin. If the air pressure in the tank is 6 lb/in.^{2} and the depth of glycerin is 10 ft, what is the pressure in lb/ft^2 at the bottom of the tank?

$$p = \gamma h + p_o = \left(78.6 \frac{lb}{ft^3}\right)(10\ ft) + \left(6 \frac{lb}{in.^2}\right)\left(\frac{144\ in.^2}{ft^2}\right)$$

$$= \underline{\underline{1650 \frac{lb}{ft^2}}}$$

2.22 On the suction side of a pump a Bourdon pressure gage reads 40-kPa vacuum. What is the corresponding absolute pressure if the local atmospheric pressure is 100 kPa (abs)?

$$p(abs) = p(gage) + p(atm)$$

$$= -40\ kPa + 100\ kPa = \underline{\underline{60\ kPa}}$$

2.26

2.26 For an atmospheric pressure of 101 kPa (abs) determine the heights of the fluid columns in barometers containing one of the following liquids: **(a)** mercury, **(b)** water, and **(c)** ethyl alcohol. Calculate the heights including the effect of vapor pressure, and compare the results with those obtained neglecting vapor pressure. Do these results support the widespread use of mercury for barometers? Why?

(Including vapor pressure)

$$p(atm) = \gamma h + p_v$$

where $p_v \sim$ vapor pressure

Thus, $$h = \frac{p(atm) - p_v}{\gamma}$$

(a) For mercury: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 1.6 \times 10^{-1} \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}} = \underline{\underline{0.759\ m}}$$

(b) For water: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 1.77 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}} = \underline{\underline{10.1\ m}}$$

(c) For ethyl alcohol: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 5.9 \times 10^3 \frac{N}{m^2}}{7.74 \times 10^3 \frac{N}{m^3}} = \underline{\underline{12.3\ m}}$$

(Without vapor pressure)

$$p(atm) = \gamma h$$

$$h = \frac{p(atm)}{\gamma}$$

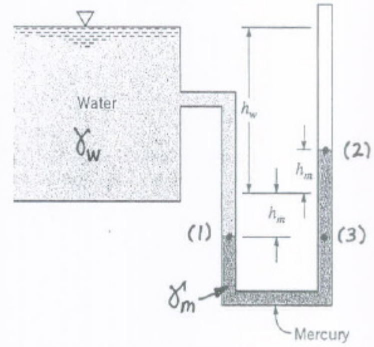
$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}} = \underline{\underline{0.759\ m}}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}} = \underline{\underline{10.3\ m}}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{7.74 \times 10^3 \frac{N}{m^3}} = \underline{\underline{13.0\ m}}$$

<u>Yes</u>. For mercury barameters the effect of vapor pressure is negligible, and the required height of the mercury column is reasonable.

2.27 A mercury manometer is connected to a large reservoir of water as shown in Fig. P2.27. Determine the ratio, h_w/h_m, of the distances h_w and h_m indicated in the figure.



■ FIGURE P2.27

$$p_1 = \gamma_w h_w + \gamma_w h_m$$

but $p_1 = p_3 = \gamma_m (2h_m)$

Thus,

$$\gamma_w h_w + \gamma_w h_m = 2\gamma_m h_m$$

or

$$(\gamma_w) h_w = (2\gamma_m - \gamma_w) h_m$$

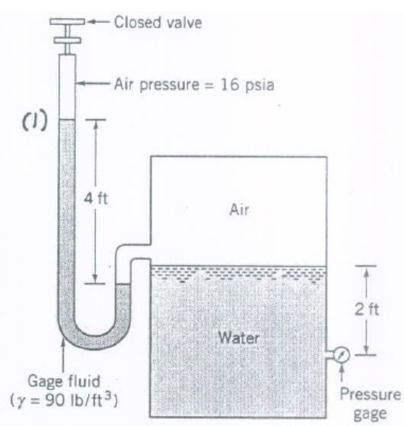
so that

$$\frac{h_w}{h_m} = \frac{(2\gamma_m - \gamma_w)}{\gamma_w} = 2\,SG_m - 1 \text{, where } SG_m = \frac{\gamma_m}{\gamma_w} = 13.56$$

Thus,

$$\frac{h_w}{h_m} = 2(13.56) - 1 = \underline{\underline{26.1}}$$

2.28 A U-tube manometer is connected to a closed tank containing air and water as shown in Fig. P2.28. At the closed end of the manometer the air pressure is 16 psia. Determine the reading on the pressure gage for a differential reading of 4 ft on the manometer. Express your answer in psi (gage). Assume standard atmospheric pressure, and neglect the weight of the air columns in the manometer.



■ FIGURE P2.28

$$p_1 + \gamma_{gf}(4\text{ ft}) + \gamma_{H_2O}(2\text{ ft}) = p_{gage}$$

Thus,

$$p_{gage} = \left(16\frac{\text{lb}}{\text{in.}^2} - 14.7\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right) + \left(90\frac{\text{lb}}{\text{ft}^3}\right)(4\text{ ft})$$

$$+ \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(2\text{ ft})$$

$$= 672\frac{\text{lb}}{\text{ft}^2} = \left(672\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}^2}{144\text{ in.}^2}\right) = \underline{\underline{4.67\text{ psi}}}$$

2.30 Two pipes are connected by a manometer as shown in Fig. P2.30. Determine the pressure difference, $p_A - p_B$, between the pipes.

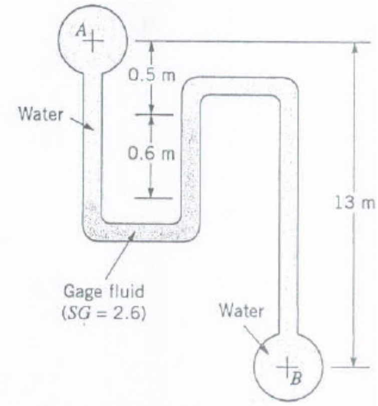


FIGURE P2.30

$$p_A + \gamma_{H_2O}(0.5m + 0.6m) - \gamma_{gf}(0.6m) + \gamma_{H_2O}(1.3m - 0.5m) = p_B$$

Thus,

$$p_A - p_B = \gamma_{gf}(0.6m) - \gamma_{H_2O}(0.5m + 0.6m + 1.3m - 0.5m)$$

$$= (2.6)(9.81\ \tfrac{kN}{m^3})(0.6m) - (9.80\ \tfrac{kN}{m^3})(1.9m)$$

$$= \underline{\underline{-\ 3.32\ kPa}}$$

2.31 A U-tube manometer is connected to a closed tank as shown in Fig. P2.31. The air pressure in the tank is 0.50 psi and the liquid in the tank is oil (γ = 54.0 lb/ft³). The pressure at point A is 2.00 psi. Determine: **(a)** the depth of oil, z, and **(b)** the differential reading, h, on the manometer.

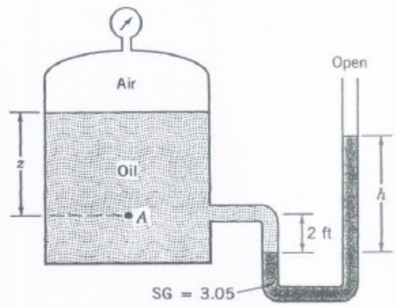


FIGURE P2.31

(a) $p_A = \gamma_{oil}\, z + p_{air}$

Thus,

$$z = \frac{p_A - p_{air}}{\gamma_{oil}} = \frac{\left(2\ \frac{lb}{in.^2} - 0.5\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{54.0\ \frac{lb}{ft^3}} = \underline{\underline{4.00\ ft}}$$

(b) $p_A + \gamma_{oil}(2ft) - (SG)(\gamma_{H_2O})\, h = 0$

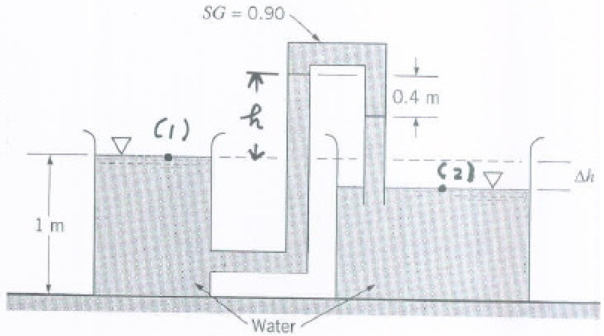
Thus,

$$h = \frac{p_A + \gamma_{oil}(2ft)}{(SG)(\gamma_{H_2O})}$$

$$= \frac{\left(2\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right) + \left(54.0\ \frac{lb}{ft^3}\right)(2ft)}{(3.05)\left(62.4\ \frac{lb}{ft^3}\right)}$$

$$= \underline{\underline{2.08\ ft}}$$

2.36 Determine the elevation difference, Δh, between the water levels in the two open tanks shown in Fig. P2.36.



■ FIGURE P2.36

$$p_1 - \gamma_{H_2O} h + (SG)\gamma_{H_2O}(0.4m) + \gamma_{H_2O}(h - 0.4m) + \gamma_{H_2O}(\Delta h) = p_2$$

Since $p_1 = p_2 = 0$

$$\Delta h = 0.4m - (0.9)(0.4m) = \underline{\underline{0.040\,m}}$$

2.41 An inverted U-tube manometer containing oil (SG = 0.8) is located between two reservoirs as shown in Fig. P2.41. The reservoir on the left, which contains carbon tetrachloride, is closed and pressurized to 8 psi. The reservoir on the right contains water and is open to the atmosphere. With the given data, determine the depth of water, h, in the right reservoir.

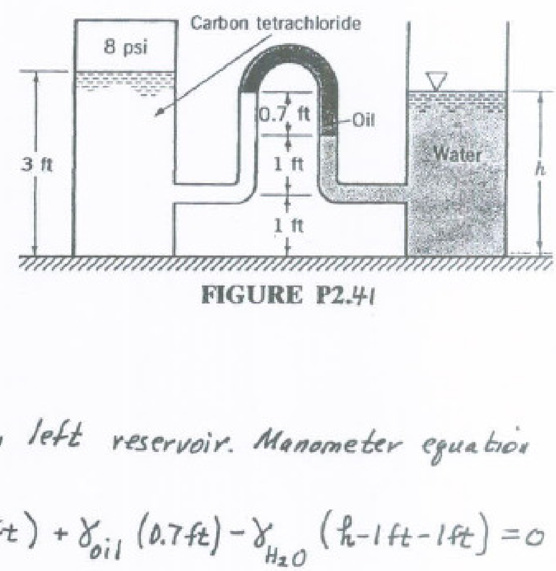


FIGURE P2.41

Let p_A be the air pressure in left reservoir. Manometer equation can be written as

$$p_A + \gamma_{CCl_4}(3ft - 1ft - 1ft - 0.7ft) + \gamma_{oil}(0.7ft) - \gamma_{H_2O}(h - 1ft - 1ft) = 0$$

so that

$$h = \frac{p_A + \gamma_{CCl_4}(0.3ft) + \gamma_{oil}(0.7ft)}{\gamma_{H_2O}} + 2ft$$

$$= \frac{\left(8\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right) + \left(99.5\frac{lb}{ft^3}\right)(0.3ft) + \left(57.0\frac{lb}{ft^3}\right)(0.7ft)}{62.4\frac{lb}{ft^3}} + 2ft$$

$$= \underline{\underline{21.6\,ft}}$$

2.42 Determine the pressure of the water in pipe A shown in Fig. P2.42 if the gage pressure of the air in the tank is 2 psi.

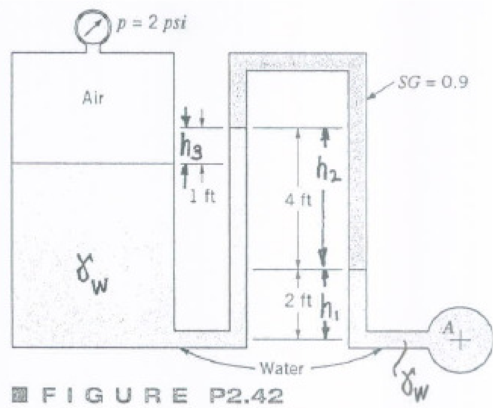


FIGURE P2.42

$$p_A - \gamma_w h_1 - (0.9\gamma_w)h_2 + \gamma_w h_3 = p_{air}$$

or

$$p_A = p_{air} + \gamma_w(h_1 + 0.9h_2 - h_3)$$

$$= 2\,\frac{lb}{in.^2}\left(\frac{144\,in.^2}{ft^2}\right) + 62.4\,\frac{lb}{ft^3}\left(\cdot\ ft + 0.9(4\,ft) - 1\,ft\right)$$

$$= 575\,\frac{lb}{ft^2}$$

2.32 For the inclined-tube manometer of Fig. P2.32 the pressure in pipe A is 0.6 psi. The fluid in both pipes A and B is water, and the gage fluid in the manometer has a specific gravity of 2.6. What is the pressure in pipe B corresponding to the differential reading shown?

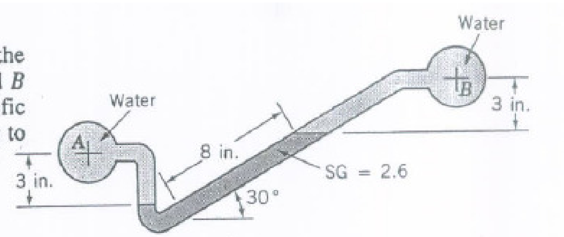


FIGURE P2.32

$$p_A + \gamma_{H_2O}\left(\frac{3}{12}\,ft\right) - \gamma_{gf}\left(\frac{8}{12}\,ft\right)\sin 30° - \gamma_{H_2O}\left(\frac{3}{12}\,ft\right) = p_B$$

(where γ_{gf} is the specific weight of the gage fluid)

Thus,

$$p_B = p_A - \gamma_{gf}\left(\frac{8}{12}\,ft\right)\sin 30°$$

$$= \left(0.6\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) - (2.6)\left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{8}{12}\,ft\right)(0.5) = 32.3\,\frac{lb}{ft^2}$$

$$= 32.3\,lb/ft^2 / 144\,in.^2/ft^2 = 0.224\ psi$$

2.43 In Fig. P2.43 pipe *A* contains gasoline ($SG = 0.7$), pipe *B* contains oil ($SG = 0.9$), and the manometer fluid is mercury. Determine the new differential reading if the pressure in pipe *A* is decreased 25 kPa, and the pressure in pipe *B* remains constant. The initial differential reading is 0.30 m as shown.

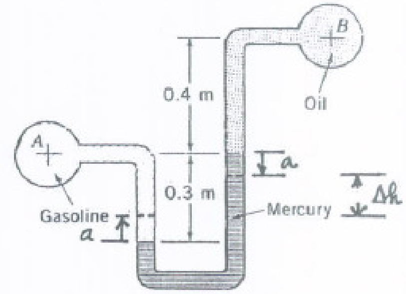


FIGURE P2.43

For the initial configuration:

$$p_A + \gamma_{gas}(0.3\,m) - \gamma_{Hg}(0.3\,m) - \gamma_{oil}(0.4\,m) = p_B \qquad (1)$$

With a decrease in p_A to p_A' gage fluid levels change as shown on figure. Thus, for final configuration:

$$p_A' + \gamma_{gas}(0.3-a) - \gamma_{Hg}(\Delta h) - \gamma_{oil}(0.4+a) = p_B \qquad (2)$$

where all lengths are in m. Subtract Eq.(2) from Eq.(1) to obtain,

$$p_A - p_A' + \gamma_{gas}(a) - \gamma_{Hg}(0.3-\Delta h) + \gamma_{oil}(a) = 0 \qquad (3)$$

Since $2a + \Delta h = 0.3$ (see figure) then

$$a = \frac{0.3-\Delta h}{2}$$

and from Eq.(3)

$$p_A - p_A' + \gamma_{gas}\left(\frac{0.3-\Delta h}{2}\right) - \gamma_{Hg}(0.3-\Delta h) + \gamma_{oil}\left(\frac{0.3-\Delta h}{2}\right) = 0$$

Thus,

$$\Delta h = \frac{p_A - p_A' + \gamma_{gas}(0.15) - \gamma_{Hg}(0.3) + \gamma_{oil}(0.15)}{-\gamma_{Hg} + \frac{\gamma_{gas}}{2} + \frac{\gamma_{oil}}{2}}$$

and with $p_A - p_A' = 25\,kPa$

$$\Delta h = \frac{25\frac{kN}{m^2} + (0.7)(9.81\frac{kN}{m^3})(0.15\,m) - (133\frac{kN}{m^3})(0.3\,m) + (0.9)(9.81\frac{kN}{m^3})(0.15\,m)}{-133\frac{kN}{m^3} + \frac{(0.7)(9.81\frac{kN}{m^3})}{2} + \frac{(0.9)(9.81\frac{kN}{m^3})}{2}}$$

$$= \underline{\underline{0.100\,m}}$$

2.44 The inclined differential manometer of Fig. P2.44 contains carbon tetrachloride. Initially the pressure differential between pipes A and B, which contain a brine ($SG = 1.1$), is zero as illustrated in the figure. It is desired that the manometer give a differential reading of 12 in. (measured along the inclined tube) for a pressure differential of 0.1 psi. Determine the required angle of inclination, θ.

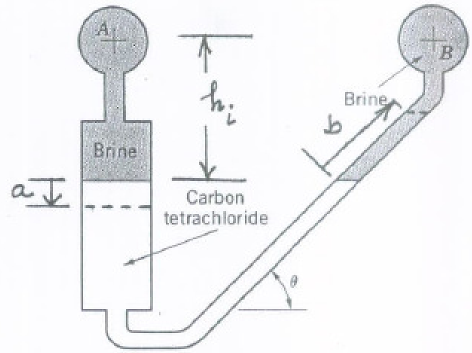


FIGURE P2.44

When $p_A - p_B$ is increased to $p_A' - p_B'$ the left column falls a distance, a, and the right column rises a distance b along the inclined tube as shown in figure. For this final configuration.

$$p_A' + \gamma_{br}(h_i + a) - \gamma_{CCl_4}(a + b\sin\theta) - \gamma_{br}(h_i - b\sin\theta) = p_B'$$

or

$$p_A' - p_B' + (\gamma_{br} - \gamma_{CCl_4})(a + b\sin\theta) = 0 \qquad (1)$$

The differential reading, Δh, along the tube is

$$\Delta h = \frac{a}{\sin\theta} + b$$

Thus, from Eq.(1)

$$p_A' - p_B' + (\gamma_{br} - \gamma_{CCl_4})(\Delta h \sin\theta) = 0$$

or

$$\sin\theta = \frac{-(p_A' - p_B')}{(\gamma_{br} - \gamma_{CCl_4})(\Delta h)}$$

and with $p_A' - p_B' = 0.1\,psi$

$$\sin\theta = \frac{-\left(0.1\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left[(1.1)\left(62.4\,\frac{lb}{ft^3}\right) - 99.5\,\frac{lb}{ft^3}\right]\left(\frac{12}{12}\,ft\right)} = 0.466$$

for $\Delta h = 12$ in.

Thus,

$$\underline{\underline{\theta = 27.8^\circ}}$$

2.45 Determine the new differential reading along the inclined leg of the mercury manometer of Fig. P2.45, if the pressure in pipe A is decreased 10 kPa and the pressure in pipe B remains unchanged. The fluid in A has a specific gravity of 0.9 and the fluid in B is water.

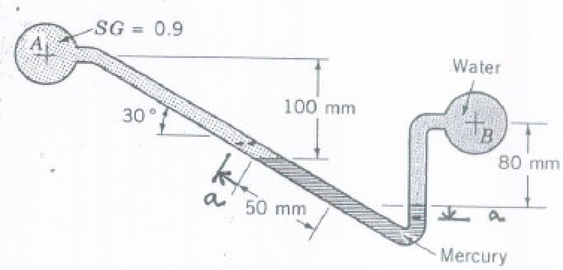


FIGURE P2.45

For the initial configuration:

$$p_A + \gamma_A(0.1) + \gamma_{Hg}(0.05 \sin 30°) - \gamma_{H_2O}(0.08) = p_B \qquad (1)$$

where all lengths are in m. When p_A decreases left column moves up a distance, a, and right column moves down a distance, a, as shown in figure. For the final configuration:

$$p_A' + \gamma_A(0.1 - a \sin 30°) + \gamma_{Hg}(a \sin 30° + 0.05 \sin 30° + a) - \gamma_{H_2O}(0.08 + a) = p_B \qquad (2)$$

where p_A' is the new pressure in pipe A.
Subtract Eq. (2) from Eq. (1) to obtain

$$p_A - p_A' + \gamma_A(a \sin 30°) - \gamma_{Hg}\, a(\sin 30° + 1) + \gamma_{H_2O}(a) = 0$$

Thus,

$$a = \frac{-(p_A - p_A')}{\gamma_A \sin 30° - \gamma_{Hg}(\sin 30° + 1) + \gamma_{H_2O}}$$

For $p_A - p_A' = 10\ kPa$

$$a = \frac{-10\ \frac{kN}{m^2}}{(0.9)(9.81\ \frac{kN}{m^3})(0.5) - (133\ \frac{kN}{m^3})(0.5+1) + 9.80\ \frac{kN}{m^3}}$$

$$= 0.0540\ m$$

New differential reading, Δh, measured along inclined tube is equal to

$$\Delta h = \frac{a}{\sin 30°} + 0.05 + a$$

$$= \frac{0.0540\ m}{0.5} + 0.05\ m + 0.0540\ m = \underline{\underline{0.212\ m}}$$

2.46 Determine the change in the elevation of the mercury in the left leg of the manometer of Fig. P2.46 as a result of an increase in pressure of 5 psi in pipe A while the pressure in pipe B remains constant.

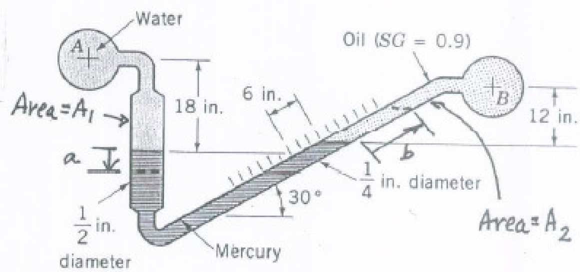


FIGURE P2.46

For the initial configuration:

$$p_A + \gamma_{H_2O}\left(\frac{18}{12}\right) - \gamma_{Hg}\left(\frac{6}{12}\sin 30^\circ\right) - \gamma_{oil}\left(\frac{12}{12}\right) = p_B \qquad (1)$$

where all lengths are in ft. When p_A increases to p_A' the left column falls by the distance, a, and the right column moves up the distance, b, as shown in the figure. For the final configuration:

$$p_A' + \gamma_{H_2O}\left(\frac{18}{12}+a\right) - \gamma_{Hg}\left(a + \frac{6}{12}\sin 30^\circ + b\sin 30^\circ\right) - \gamma_{oil}\left(\frac{12}{12} - b\sin 30^\circ\right) = p_B \qquad (2)$$

Subtract Eq. (1) from Eq. (2) to obtain

$$p_A' - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + b\sin 30^\circ) + \gamma_{oil}(b\sin 30^\circ) = 0 \qquad (3)$$

Since the volume of liquid must be constant $A_1 a = A_2 b$, or

$$\left(\tfrac{1}{2}\text{ in.}\right)^2 a = \left(\tfrac{1}{4}\text{ in.}\right)^2 b$$

so that

$$b = 4a$$

Thus, Eq. (3) can be written as

$$p_A' - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + 4a\sin 30^\circ) + \gamma_{oil}(4a\sin 30^\circ) = 0$$

and

$$a = \frac{-(p_A' - p_A)}{\gamma_{H_2O} - \gamma_{Hg}(3) + \gamma_{oil}(2)} = \frac{-\left(5\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{62.4\,\frac{lb}{ft^3} - \left(847\,\frac{lb}{ft^3}\right)(3) + (0.9)\left(62.4\,\frac{lb}{ft^3}\right)(2)}$$

$$= \underline{\underline{0.304\text{ ft (down)}}}$$

2.52 A piston having a cross-sectional area of 0.07 m² is located in a cylinder containing water as shown in Fig. P2.52. An open U-tube manometer is connected to the cylinder as shown. For $h_1 = 60$ mm and $h = 100$ mm, what is the value of the applied force, P, acting on the piston? The weight of the piston is negligible.

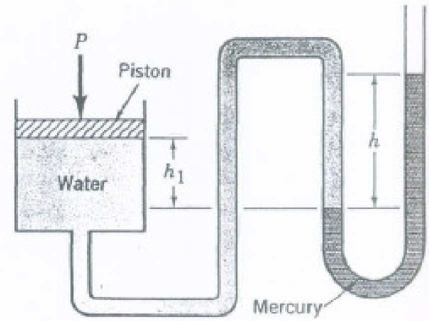


FIGURE P2.52

For equilibrium, $P = p_p A_p$ where p_p is the pressure acting on piston and A_p is the area of the piston. Also,

$$p_p + \gamma_{H_2O}\, h_1 - \gamma_{Hg}\, h = 0$$

or

$$p_p = \gamma_{Hg}\, h - \gamma_{H_2O}\, h_1$$

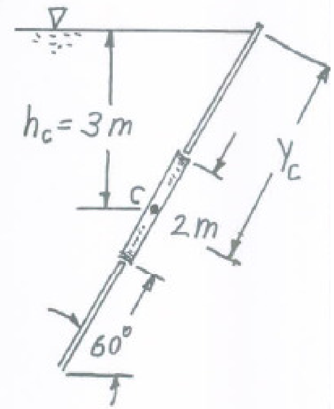
$$= \left(133 \frac{kN}{m^3}\right)(0.100\,m) - \left(9.80 \frac{kN}{m^3}\right)(0.060\,m)$$

$$= 12.7 \frac{kN}{m^2}$$

Thus,

$$P = \left(12.7 \times 10^3 \frac{N}{m^2}\right)(0.07\,m^2) = \underline{\underline{889\,N}}$$

2.54 A circular 2-m-diameter gate is located on the sloping side of a swimming pool. The side of the pool is oriented 60° relative to the horizontal bottom, and the center of the gate is located 3 m below the water surface. Determine the magnitude of the water force acting on the gate and the point through which it acts.



$F_R = p_c A = \gamma h_c A$, where $h_c = 3m$

Thus;

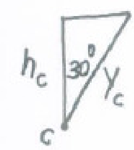
$$F_R = (9.8\,\frac{kN}{m^3})(3m)(\frac{\pi}{4}(2m)^2) = \underline{\underline{94.2\ kN}}$$

Also,

$$y_R - Y_c = \frac{I_{xc}}{Y_c A}, \quad \text{where for a circle } I_{xc} = \frac{\pi R^4}{4} = \frac{\pi (1m)^4}{4} = \frac{\pi}{4}\,m^4$$

and $\cos 30° = \frac{h_c}{Y_c}$ so that

$$Y_c = \frac{h_c}{\cos 30°} = \frac{3\ m}{\cos 30°} = 3.46\,m$$



Hence,

$$y_R - Y_c = \frac{I_{xc}}{Y_c A} = \frac{\frac{\pi}{4}\,m^4}{(3.46m)\,\frac{\pi}{4}(2m)^2} = \underline{\underline{0.0723\ m}}$$

Thus, the resultant force acts normal to the gate and 0.0723 m from the centroid, along the gate.

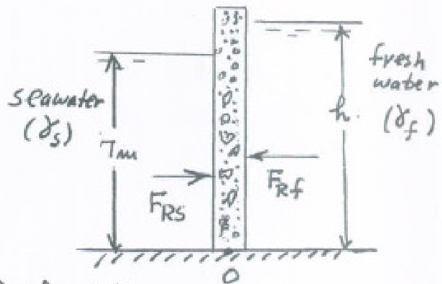
2.59 A long, vertical wall separates seawater from freshwater. If the seawater stands at a depth of 7 m, what depth of freshwater is required to give a zero resultant force on the wall? When the resultant force is zero will the moment due to the fluid forces be zero? Explain.

For a zero resultant force

$$F_{RS} = F_{Rf}$$

or

$$\gamma_s h_{cs} A_s = \gamma_f h_{cf} A_f$$



Thus, for a unit length of wall

$$\left(10.1 \frac{kN}{m^3}\right)\left(\frac{7m}{2}\right)(7m \times 1m) = \left(9.80 \frac{kN}{m^3}\right)\left(\frac{h}{2} m\right)(h \times 1m)$$

so that

$$\underline{\underline{h = 7.11 \ m}}$$

In order for moment to be zero, F_{RS} and F_{Rf} must be collinear.

For F_{RS}:
$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(1m)(7m)^3}{\left(\frac{7}{2}m\right)(7m \times 1m)} + \frac{7}{2} m = 4.67 m$$

Similarly for F_{Rf}:

$$y_R = \frac{\frac{1}{12}(1m)(7.11m)^3}{\left(\frac{7.11}{2}m\right)(7.11m \times 1m)} + \frac{7.11}{2} m = 4.74 m$$

Thus, the distance to F_{RS} from the bottom (point O) is $7m - 4.67m = 2.33 m$. For F_{Rf} this distance is $7.11m - 4.74m = 2.37 m$. The forces are not collinear. $\underline{\underline{\text{No.}}}$

2.61

2.61 A homogeneous, 4-ft-wide, 8-ft-long rectangular gate weighing 800 lb is held in place by a horizontal flexible cable as shown in Fig. P2.61 Water acts against the gate which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.

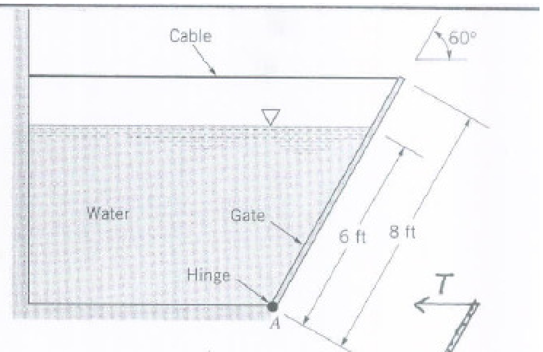
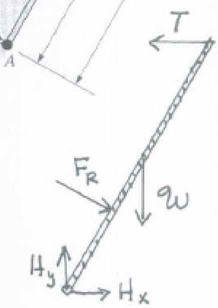


FIGURE P2.61



$F_R = \gamma h_c A \quad$ where $h_c = \left(\frac{6 ft}{2}\right) \sin 60°$

Thus,

$$F_R = \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{6 ft}{2}\right)(\sin 60°)(6 ft \times 4 ft)$$

$$= 3890 \text{ lb}$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where } y_c = 3 ft$$

so that

$$y_R = \frac{\frac{1}{12}(4 ft)(6 ft)^3}{(3 ft)(6 ft \times 4 ft)} + 3 ft = 4.0 ft$$

For equilibrium,

$$\sum M_H = 0$$

and

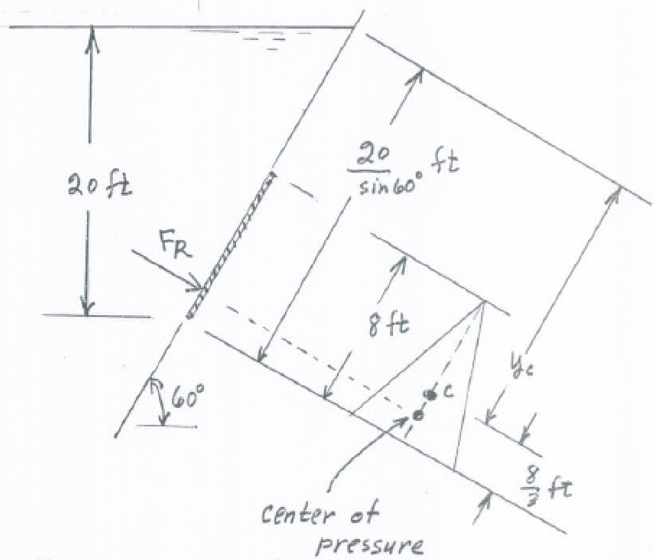
$$T (8 ft)(\sin 60°) = \mathcal{W} (4 ft)(\cos 60°) + F_R (2 ft)$$

$$T = \frac{(800 lb)(4 ft)(\cos 60°) + (3890 lb)(2 ft)}{(8 ft)(\sin 60°)}$$

$$= \underline{\underline{1350 \text{ lb}}}$$

2.63

2.63 An area in the form of an isosceles triangle with a base width of 6 ft and an altitude of 8 ft lies in the plane forming one wall of a tank which contains a liquid having a specific weight of 79.8 lb/ft³. The side slopes upward making an angle of 60° with the horizontal. The base of the triangle is horizontal and the vertex is above the base. Determine the resultant force the fluid exerts on the area when the fluid depth is 20 ft above the base of the triangular area. Show, with the aid of a sketch, where the center of pressure is located.



$$y_c = \left(\frac{20}{\sin 60°}\right) ft - \left(\frac{8}{3}\right) ft$$

$$= 20.43 \text{ ft}$$

$$h_c = y_c \sin 60°$$

$$F_R = \gamma h_c A = \left(79.8 \frac{lb}{ft^3}\right)\left[(20.43 \text{ ft}) \sin 60°\right]\left(\frac{1}{2}\right)(6 \text{ ft} \times 8 \text{ ft})$$

$$= \underline{\underline{33{,}900 \text{ lb}}}$$

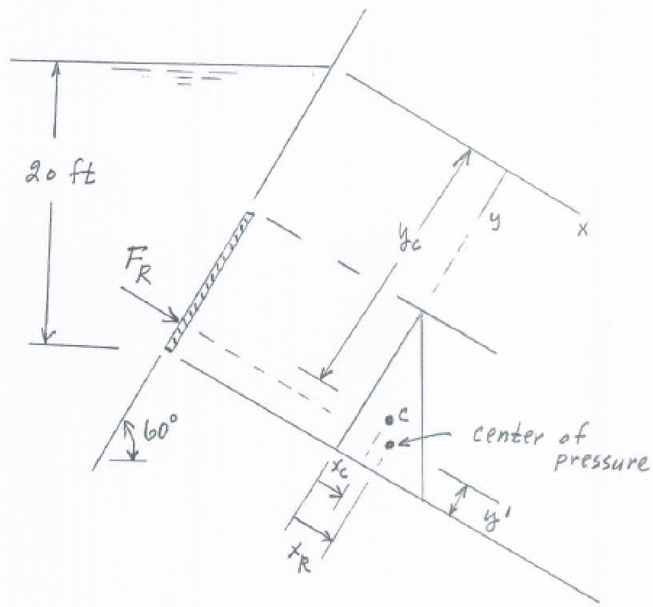
$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where} \quad I_{xc} = \frac{1}{36}(6ft)(8ft)^3$$

Thus,

$$y_R = \frac{\frac{1}{36}(6ft)(8ft)^3}{(20.43ft)(\frac{1}{2})(6ft \times 8ft)} + 20.43 \text{ ft} = 20.6 \text{ ft}$$

The force, F_R, acts through the center of pressure which is located a distance of $\frac{20}{\sin 60°}$ ft - 20.6 ft = 2.49 ft above the base of the triangle as shown in sketch.

2.64 Solve Problem 2.63 if the isosceles triangle is replaced with a right triangle having the same base width and altitude as the isosceles triangle.



$F_R = \underline{\underline{33,900\ lb}}$

$y' = \underline{\underline{2.49\ ft}}$

(see solution to Problem 2.63)

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \qquad \text{(Eq. 2.20)}$$

where

$$I_{xyc} = \frac{(6\ ft)^2(8\ ft)^2}{72} = 32\ ft^4 \qquad \text{(see Fig. 2.18 d)}$$

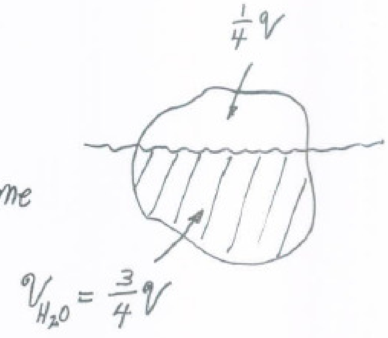
and $y_c = 20.43\ ft$ (see solution to Problem 2.63)

Thus,

$$x_R = \frac{32\ ft^4}{(20.43\ ft)(\frac{1}{2})(6\ ft \times 8\ ft)} + \frac{6}{3}\ ft = \underline{\underline{2.07\ ft}}$$

The force, F_R, acts through the center of pressure with coordinates $x_R = 2.07\ ft$ and $y' = 2.49\ ft$ (see sketch).

2.97 A freshly cut log floats with one fourth of its volume protruding above the water surface. Determine the specific weight of the log.



$\mathcal{V}$ = log volume

$F_B = W$ or

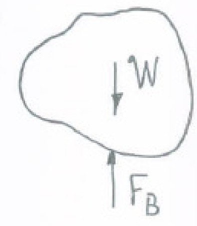
$\gamma_{H_2O} \mathcal{V}_{H_2O} = \gamma_{log} \mathcal{V}$

Thus,

$$\gamma_{log} = \gamma_{H_2O} \frac{\mathcal{V}_{H_2O}}{\mathcal{V}} = \gamma_{H_2O} \frac{\frac{3}{4}\mathcal{V}}{\mathcal{V}}$$

or

$$\gamma_{log} = \frac{3}{4}\gamma_{H_2O} = \frac{3}{4}\left(62.4 \frac{lb}{ft^3}\right) = \underline{\underline{46.8 \frac{lb}{ft^3}}}$$



2.98 A river barge, whose cross section is approximately rectangular, carries a load of grain. The barge is 28 ft wide and 90 ft long. When unloaded its draft (depth of submergence) is 5 ft, and with the load of grain the draft is 7 ft. Determine: **(a)** the unloaded weight of the barge, and **(b)** the weight of the grain.

(a) For equilibrium,

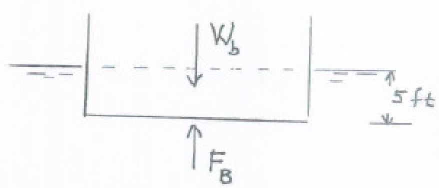
$$\Sigma F_{vertical} = 0$$

so that

$$W_b = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$= \left(62.4 \frac{lb}{ft^3}\right)(5\ ft \times 28\ ft \times 90\ ft)$$

$$= \underline{\underline{786{,}000\ lb}}$$



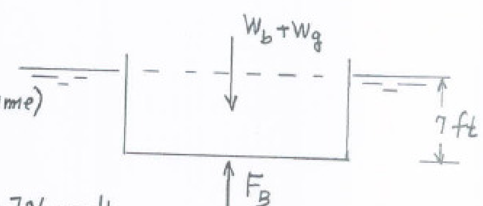
$W_b \sim$ weight of barge (unloaded)

(b) $\Sigma F_{vertical} = 0$

$$W_b + W_g = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$W_g = \left(62.4 \frac{lb}{ft^3}\right)(7 ft \times 28 ft \times 90 ft) - 786{,}000\ lb$$

$$= \underline{\underline{315{,}000\ lb}}$$



$W_g \sim$ weight of grain

2.99 A tank of cross-sectional area A is filled with a liquid of specific weight γ_1 as shown in Fig. P2.99*a*. Show that when a cylinder of specific weight γ_2 and volume $\forall$ is floated in the liquid (see Fig. P2.99*b*), the liquid level rises by an amount $\Delta h = (\gamma_2/\gamma_1)\forall/A$.

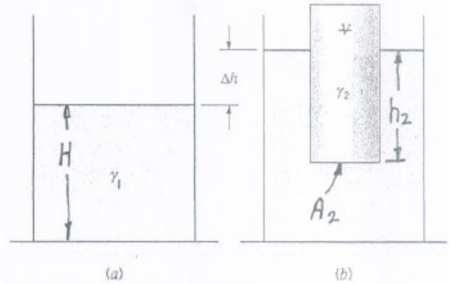


FIGURE P2.99

W = weight of cylinder $= \gamma_2 \forall$

For equilibrium,

W = weight of liquid displaced $= \gamma_1 h_2 A_2 = \gamma_1 \forall_2$ where $\forall_2 = h_2 A_2$

Thus,

$\gamma_2 \forall = \gamma_1 \forall_2$, or

$$\forall_2 = \frac{\gamma_2}{\gamma_1}\forall$$

However, the final volume within the tank is equal to the initial volume plus the volume, $\forall_2$, of the cylinder that is submerged. That is,

$$(H+\Delta h)A = HA + \forall_2$$

or

$$\Delta h = \frac{\forall_2}{A} = \underline{\underline{\frac{\gamma_2}{\gamma_1}\frac{\forall}{A}}}$$

2.103 An irregularly shaped piece of a solid material weighs 8.05 lb in air and 5.26 lb when completely submerged in water. Determine the density of the material.

W (in air) $= \rho g \times$ (volume) where $\rho \sim$ density of material

W (in water) $= \rho g \times$ (volume) $-$ buoyant force

$= \rho g \times$ (volume) $- \rho_{H_2O}\, g \times$ (volume)

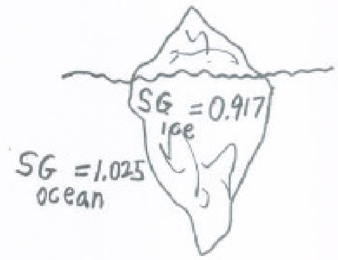
Thus,

$$\frac{W\text{ (in air)}}{W\text{ (in water)}} = \frac{\rho}{\rho - \rho_{H_2O}} = \frac{1}{1 - \frac{\rho_{H_2O}}{\rho}}$$

or

$$\rho = \frac{\rho_{H_2O}}{1 - \frac{W\text{ (in water)}}{W\text{ (in air)}}} = \frac{1.94\ \frac{\text{slugs}}{\text{ft}^3}}{1 - \frac{5.26\ \text{lb}}{8.05\ \text{lb}}} = \underline{\underline{5.60\ \frac{\text{slugs}}{\text{ft}^3}}}$$

2.108 An ice berg (specific gravity 0.917) floats in the ocean (specific gravity 1.025). What percent of the volume of the iceberg is under water?



For equilibrium,

$\mathcal{W}$ = weight of iceberg = F_B = buoyant force

or

$\forall_{ice}\,\gamma_{ice} = \forall_{sub}\,\gamma_{ocean}$, where $\forall_{sub}$ = volume of ice submerged.

Thus,

$$\frac{\forall_{sub}}{\forall_{ice}} = \frac{\gamma_{ice}}{\gamma_{ocean}} = \frac{SG_{ice}}{SG_{ocean}} = \frac{0.917}{1.025} = 0.895 = \underline{\underline{89.5\,\%}}$$

1.95 Small droplets of carbon tetrachloride at 68 °F are formed with a spray nozzle. If the average diameter of the droplets is 200 μm what is the difference in pressure between the inside and outside of the droplets?

$$p = \frac{2\sigma}{R} \qquad \text{(Eq. 1.21)}$$

Since $\sigma = 2.69 \times 10^{-2}\ \frac{N}{m}$ at $68°F\,(=20°C)$,

$$p = \frac{2\,(2.69 \times 10^{-2}\ \frac{N}{m})}{100 \times 10^{-6}\ m} = \underline{\underline{538\ Pa}}$$

1.103 (See Fluids in the News article titled "Walking on water," Section 1.9.) **(a)** The water strider bug shown in Fig. P1.103 is supported on the surface of a pond by surface tension acting along the interface between the water and the bug's legs. Determine the minimum length of this interface needed to support the bug. Assume the bug weighs 10^{-4} N and the surface tension force acts vertically upwards. **(b)** Repeat part (a) if surface tension were to support a person weighing 750 N.

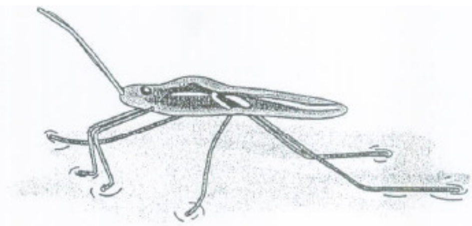
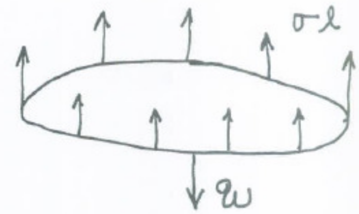


FIGURE P1.103



For equilibrium,

$$W = \sigma \ell$$

$W \sim$ weight

$\sigma \sim$ surface tension

$\ell \sim$ length of interface

(a) $$\ell = \frac{W}{\sigma} = \frac{10^{-4}\ N}{7.34 \times 10^{-2}\ \frac{N}{m}}$$

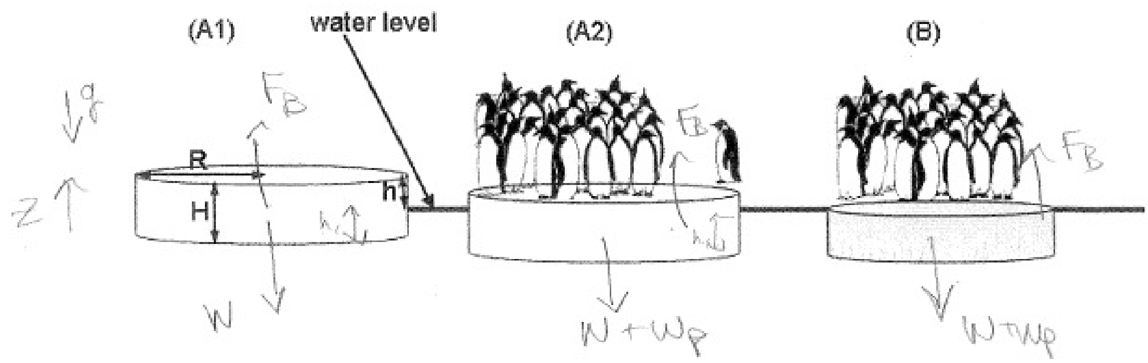
$$= 1.36 \times 10^{-3}\ m$$

$$= \left(1.36 \times 10^{-3}\ m\right)\left(10^{3}\ \frac{mm}{m}\right) = \underline{\underline{1.36\ mm}}$$

(b) $$\ell = \frac{750\ N}{7.34 \times 10^{-2}\ \frac{N}{m}} = \underline{\underline{1.02 \times 10^{4}\ m}} \quad (6.34\ mi\ !!)$$

Extra problems on buoyancy
Problem 1:
A cylindrical slab of ice (density 900kg/m^3) of circular cross section (radius R=2.0m) and height H=20cm floats in sea water (density 1030kg/m^3).

(a) A group of N=50 penguins (mass m=5.0kg each) decides this is a good place to rest and they hop onto the slab. Determine the height h by which the ice slab protrudes above water (i) without the penguins; (ii) after the penguins have hopped on.



A1) $F_B = \rho_f g V_{disp} = \rho_f g h_1 \pi R^2 = W = \rho_{ice} g H \pi R^2$

$$\rho_f h_1 = \rho_{ice} H$$

$$h_1 = \frac{\rho_{ice} H}{\rho_f} = \frac{(900\ \frac{kg}{m^3})(0.2m)}{1030\ kg/m^3} = 0.175m$$

$$h = H - h_1 = .2 - 0.17\ m = 0.0252\ \text{meters}$$

A2)

$$\rho_f g h_1 \pi R^2 = \rho_{ice} g H \pi R^2 + (250\ kg)(g)$$

$$h_1 = \underbrace{\frac{\rho_{ice} H}{\rho_f}}_{= 0.17 \text{ meters from part A1}} + \underbrace{\frac{250\ kg}{\rho_f \pi R^2}}_{\frac{250\ kg}{1030\frac{kg}{m^3} \pi 4 m^2}} = 0.175m + 0.0193m$$

$$= 0.1943 \text{ meters}$$

$$h = H - h_1 = 0.2m - 0.1943 \text{ meters} = 5.7e^{-3} m$$

$$= .0057m = 5.7\ mm$$

18) The key insight here is that at the instant the (iceberg + pengs) goes under,

$$\underset{\substack{\text{water}\\\text{displaced}}}{\forall} = \forall_{ice} = \underset{\text{(after melting)}}{\forall_{new}}$$

It does not matter how the volume melted.

$$\underbrace{\rho_w \cancel{g} \forall_{new}}_{\substack{F_B \text{ on}\\ \text{melted ice}}} = \underbrace{\rho_{ice} \cancel{g} \forall_{new}}_{W_{\substack{\text{melted}\\ \text{ice}}}} + \underbrace{(250 \text{ kg})\cancel{(g)}}_{W_{pengs}}$$

$$\rho_w \forall_{new} - \rho_{ice} \forall_{new} = 250 \text{ kg}$$

$$\forall_{new} (\rho_w - \rho_{ice}) = 250 \text{ kg}$$

$$\forall_{new} \left(1030 \text{ kg/m}^3 - 900 \text{ kg/m}^3 \right) = 250 \text{ kg}$$

$$\forall_{new} = \frac{250 \text{ kg}}{130 \text{ kg/m}^3} = 1.92 \text{ m}^3$$

That's the new volume of the iceberg after enough has melted to just barely submerge it.

How much has to melt to attain $\forall_{new}$?

$$\forall_{orig} - \forall_{new} = \pi R^2 H \text{ m}^3 - 1.92 \text{ m}^3$$
$$= .8\pi \text{ m}^3 - 1.92 \text{ m}^3 = 0.59 \text{ m}^3$$

How long does it take to melt that volume?

$$0.59 \text{ m}^3 = \left(\frac{500 \text{ cm}^3}{\text{hour}}\right)\left(\frac{1\text{m}}{100 \text{ cm}}\right)^3 (\text{t hours}) \implies \text{t} = \frac{0.59 \text{ m}^3}{.0005 \frac{\text{m}^3}{\text{hour}}} = 1180 \text{ hours}$$

Problem 2: A hot-air balloon weighing 80 kg has a capacity of 1200 m^3. If it is filled with helium, how great a payload can it support? The density of helium is 0.18 kg/ m^3 and the density of air is 1.30 kg/ m^3. Express your answer in Newtons.

First, determine the total mass that can be lifted by the helium balloon.

Mass lifted = mass of the volume of fluid displaced

Mass of displaced air = (1200 m^3)(1.3 kg/m^3) = 1560 kg

The total amount that can be lifted = 1560 kg (including balloon, helium and payload)

Now, the total amount that can be lifted must include the mass of the balloon and the mass of the helium itself, so we need to subtract those:

$$1560 \text{ kg} - 80 \text{ kg} - (1200 \text{ m}^3)*(0.18 \text{ kg/m}^3) = 1264 \text{ kg}$$

Converting to Newtons yields:

$F = (1264 \text{ kg}) * (9.8 \text{ m/s}^2) = 12{,}387$ Newtons

Problem 3 Sea turtles use their lungs to control their buoyancy. Here are 2 facts about turtles:

FACT ONE:

Before a turtle dives underwater, it inhales some air into its lungs. During the turtle's dive, the air volume in the lungs changes with increasing water pressure according to the ideal gas law:

$$P = \rho RT$$

This means that the total body density of the turtle changes with depth.

FACT TWO:

Turtles prefer to swim at the depth where they experience "neutral buoyancy," that is, where the vertical forces on the turtle are exactly zero.

Part A) Clearly identify each variable in the ideal gas law, including units

$$P = \rho \; R \; T$$

absolute pressure in Pascals

density in $\frac{kg}{m^3}$

gas constant

absolute temperature in °K

UNITS FOR R

$$\boxed{\left(\frac{\cancel{kg}}{m\,s^2}\right)\left(\frac{m^3}{\cancel{kg}}\right)\left(\frac{1}{°K}\right) = \frac{m^2}{s^2\,°K}}$$

Part B) Using Fact One and Fact Two, write an expression for the depth h at which a turtle will prefer to swim. Your expression may involve some or all of the following variables:

M_T = Mass of the turtle without air
V_T = Volume of the turtle without air
M_{air} = Mass of the air the turtle takes in before diving
V_{air} = Volume of the air the turtle takes in before diving
ρ_w = density of sea water
g = the acceleration due to gravity
The variables R and T in the ideal gas law. <u>You can assume T is constant with depth.</u>

Neutral buoyancy when $\sum F_z = 0$

$$F_W = (m_{air} + m_T)g$$ ← weight does not change with depth

$$F_B = (\forall_T + \forall_{air})\rho_w g$$

$$P_{air} = \rho_{air} RT = \frac{m_{air}}{\forall_{air}} RT$$

$$P_{air} = \rho_w g h \qquad \forall_{air} = \frac{m_{air} RT}{\rho_w g h}$$

$$F_B = (\forall_T + \forall_{air})\rho_w g$$

$$= \left(\forall_T + \frac{m_{air} RT}{\rho_w g h}\right)\rho_w g$$

$$F_B = F_W \Rightarrow$$

$$(m_{air} + m_T)\cancel{g} = \left(\forall_T \rho_w + \frac{m_{air} RT}{gh}\right)\cancel{g}$$

$$(m_{air} + m_T) - V_T \rho_w = \frac{m_{air} RT}{gh}$$

$$\frac{1}{h} = \frac{(m_{air} + m_T - V_T\rho_w)g}{m_{air} RT} \qquad \left[\frac{\frac{kg\,m}{s^2}}{(kg)\left(\frac{m^2}{s^2\,°K}\right)\left(\frac{°K}{1}\right)}\right]^{-1} \text{units check}$$

$$\boxed{h = \left(\frac{m_{air} RT}{(m_{air} + m_T - V_T\rho_w)g}\right) \text{meters}}$$

$$\left[\left(\frac{kg\,m}{s^2}\right)\left(\frac{s^2}{kg\,m^2}\right)\right]^{-1}$$

$$= \text{meters} \checkmark$$