LAPLACE TRANSFORMS

Introduction to Laplace transforms

2st semester 2019-2020

Instructor: Dr. Vu Quang Huy

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Introduction

Why it is important to understand: **Introduction to Laplace transforms**

- **The Laplace transform is a very powerful mathematical tool applied in various areas of engineering and science. With the increasing complexity of engineering problems, Laplace transforms help in solving complex problems with a very simple approach; the transform is an integral transform method which is particularly useful in solving linear ordinary differential equations. It has very wide applications in various areas of physics, electrical engineering, control engineering, optics, mathematics and signal processing. This chapter just gets us started in understanding some standard Laplace transforms.**

At the end of this chapter, you should be able to:

- define a Laplace transform
- recognise common notations used for the Laplace transform
- derive Laplace transforms of elementary functions
- use a standard list of Laplace transforms to determine the transform of common functions

67.1 Introduction

The solution of most electrical circuit problems can be reduced ultimately to the solution of differential equations. The use of **Laplace∗ transforms** provides an alternative method for solving linear differential equations.

67.2 Definition of a Laplace transform

The Laplace transform of the function $f(t)$ is defined by the integral $\int_0^{\infty} e^{-st} f(t)\,dt$, where s is a parameter assumed to be a real number.

(i) $\mathcal{L}\{f(t)\}$ or $L\{f(t)\}$

(ii) $\mathcal{L}(f)$ or Lf

(iii) $\overline{f}(s)$ or $f(s)$

Common notations used for the Laplace transform

There are various commonly used notations for the Laplace transform of $f(t)$ and these include:

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67.3 Linearity property

From equation (1),

$$\mathcal{L}\{k f(t)\} = \int_0^\infty e^{-st} k\, f(t)\, dt$$

$$= k \int_0^\infty e^{-st}\, f(t)\, dt$$

i.e. $\mathcal{L}\{k\, f(t)\} = k\mathcal{L}\{f(t)\}$ (2)

where k is any constant.

Similarly,

$$\mathcal{L}\{a\, f(t) + bg(t)\} = \int_0^\infty e^{-st}(a\, f(t) + bg(t))\, dt$$

$$= a \int_0^\infty e^{-st}\, f(t)\, dt + b \int_0^\infty e^{-st} g(t)\, dt$$

i.e. $\mathcal{L}\{a f(t) + bg(t)\} = a\mathcal{L}\{ f(t)\} + b\mathcal{L}\{g(t)\}$, (3)

where a and b are any real constants.

The Laplace transform is termed a **linear operator**

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where a and b are any real constants.
The Laplace transform is termed a **linear operator**

67.4 Laplace transforms of elementary functions

(a) $f(t) = 1$. From equation (1),

$$\mathcal{L}\{1\} = \int_0^\infty e^{-st}(1)\,dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= -\frac{1}{s}[e^{-s(\infty)} - e^0] = -\frac{1}{s}[0 - 1]$$

$$= \frac{1}{s} \text{ (provided } s > 0)$$

(b) $f(t) = k$. From equation (2),

$$\mathcal{L}\{k\} = k\mathcal{L}\{1\}$$

Hence $\mathcal{L}\{k\} = k\left(\frac{1}{s}\right) = \frac{k}{s}$, from (a) above.

67.4 Laplace transforms of elementary functions

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67.5 Worked problems on standard Laplace transforms

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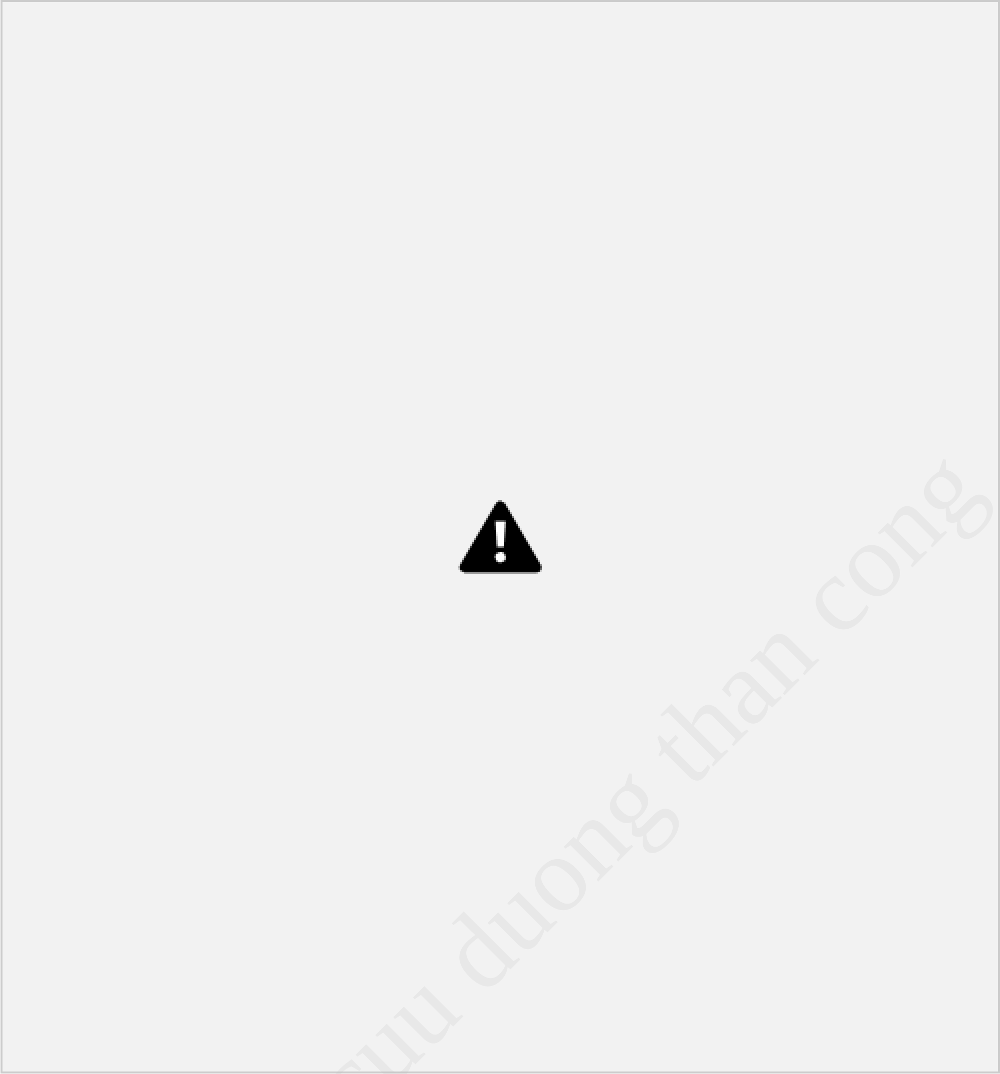

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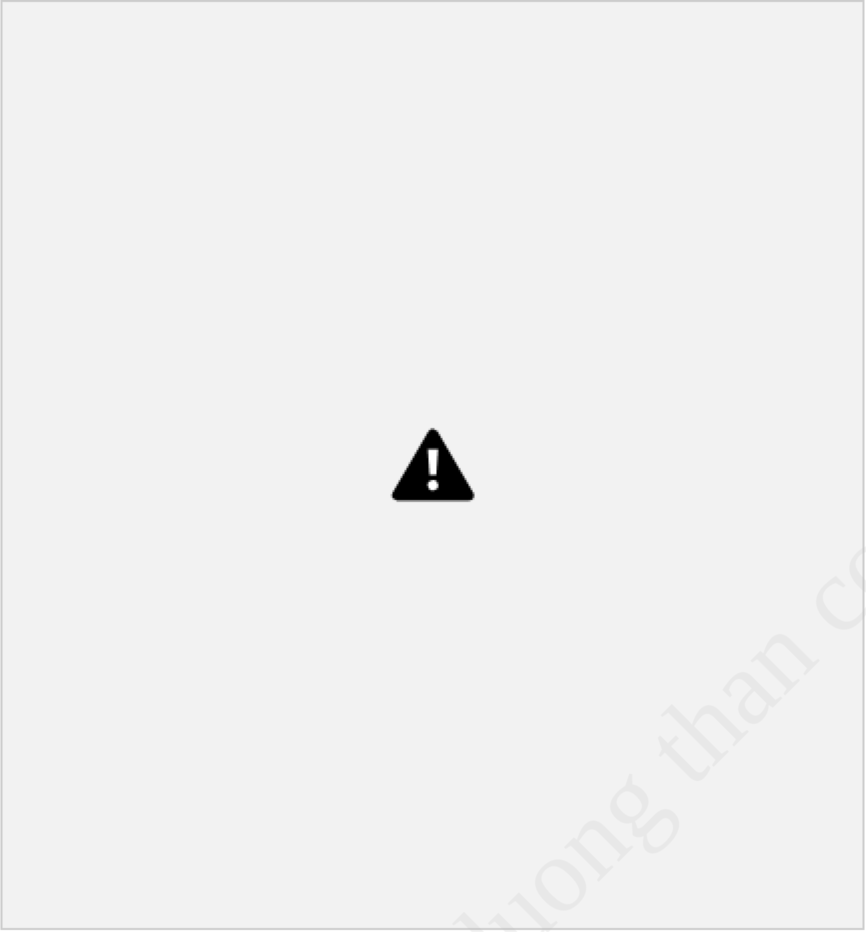
67.5 Worked problems on standard Laplace transforms



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67.5 Worked problems on standard Laplace transforms

LAPLACE TRANSFORMS

Properties of Laplace transforms

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Introduction

Why it is important to understand: **Properties of Laplace transforms**

- **As stated in the preceding chapter, the Laplace transform is a widely used integral transform with many applications in engineering, where it is used for analysis of linear time-invariant systems such as electrical circuits, harmonic oscillators, optical devices, and mechanical systems. The Laplace transform is also a valuable tool in solving differential equations, such as in electronic circuits, and in feedback control systems, such as in stability and control of aircraft systems.**
- **This chapter considers further transforms together with the Laplace transform of derivatives that are needed when solving differential equations.**

At the end of this chapter, you should be able to:

67.1 Introduction

The solution of most electrical circuit problems can be reduced ultimately to the solution of differential equations. The use of **Laplace*** **transforms** provides an alternative method for solving linear differential equations.

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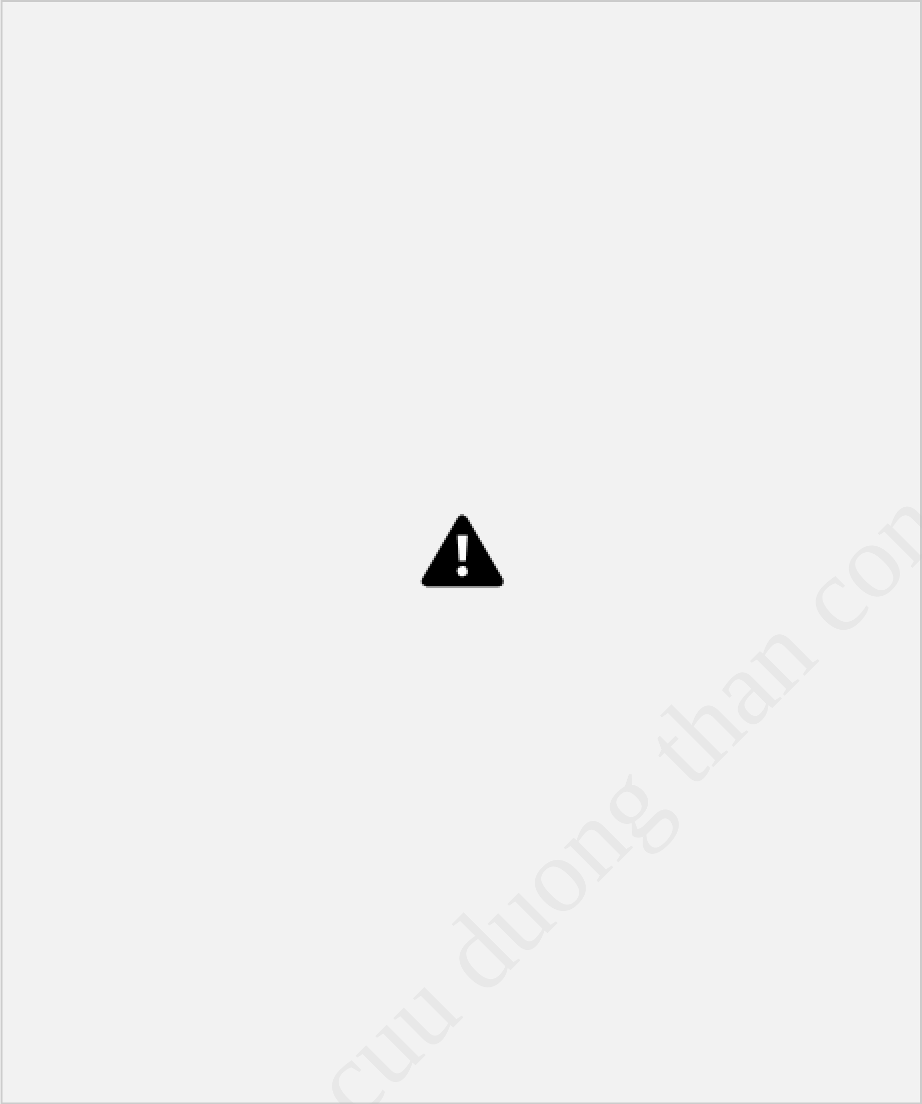

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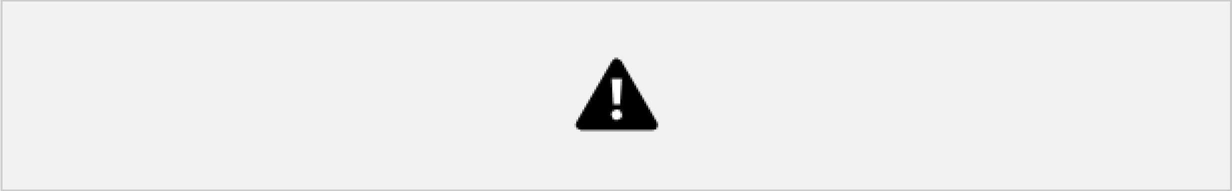

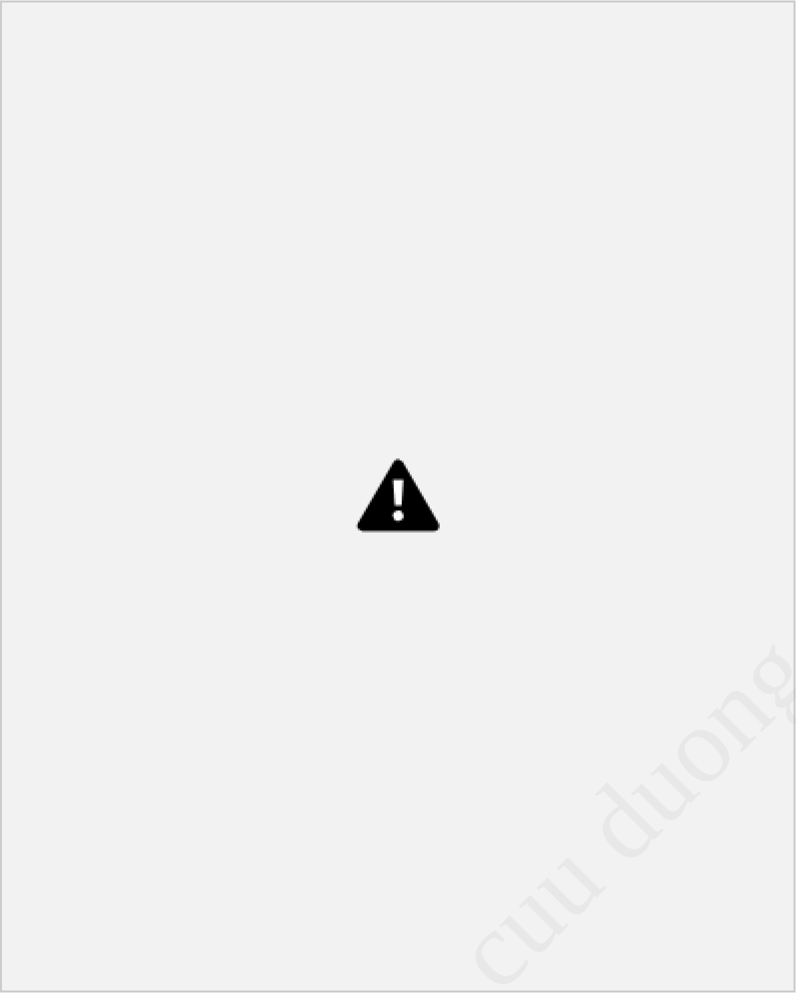

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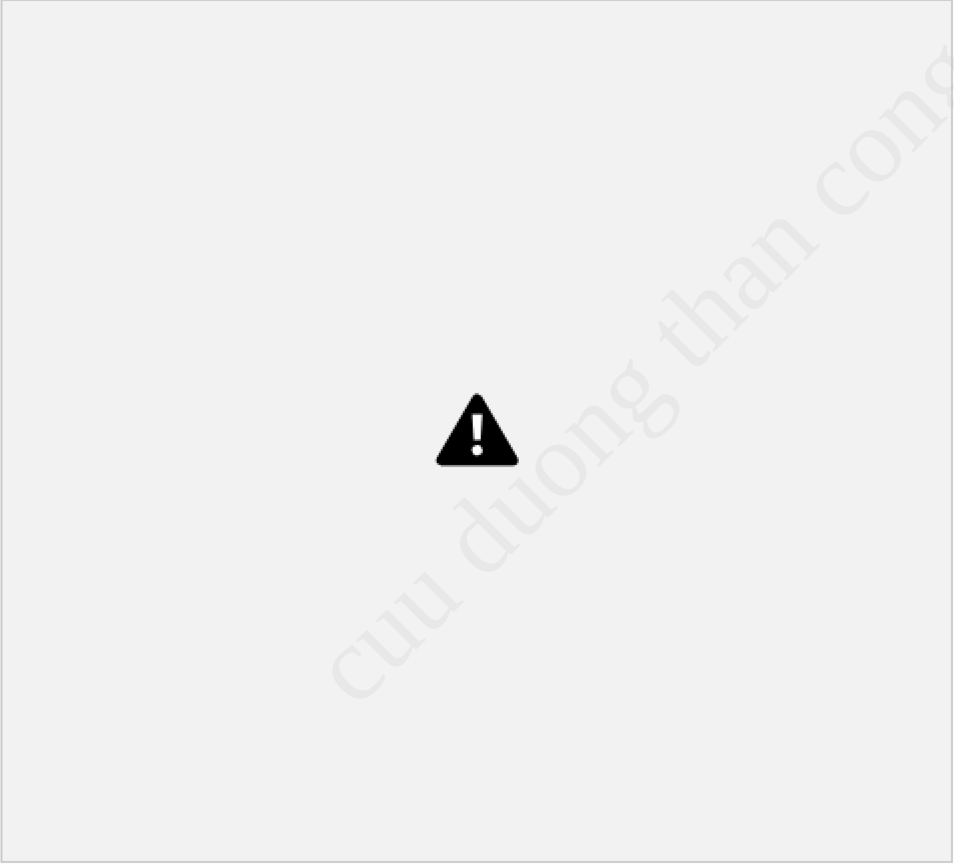

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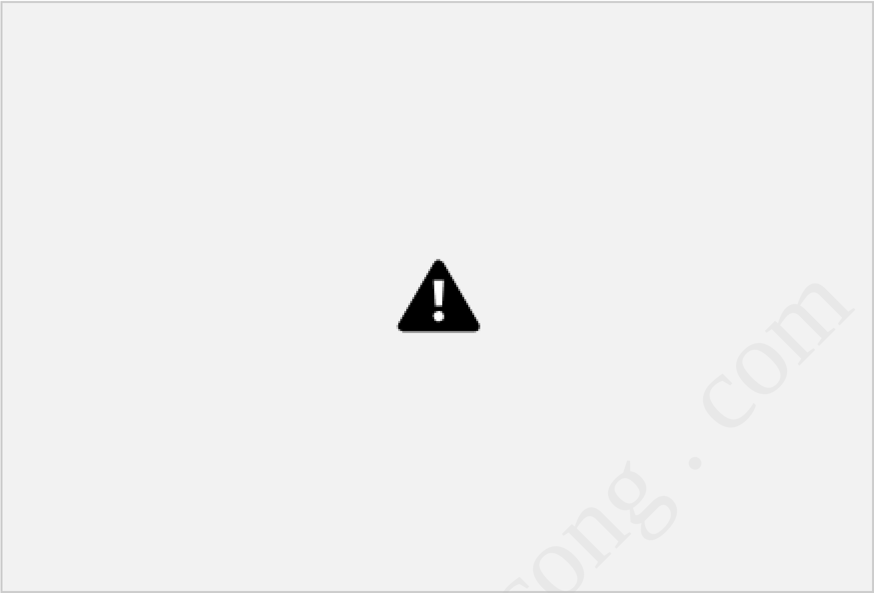

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68.4 The initial and final value theorems

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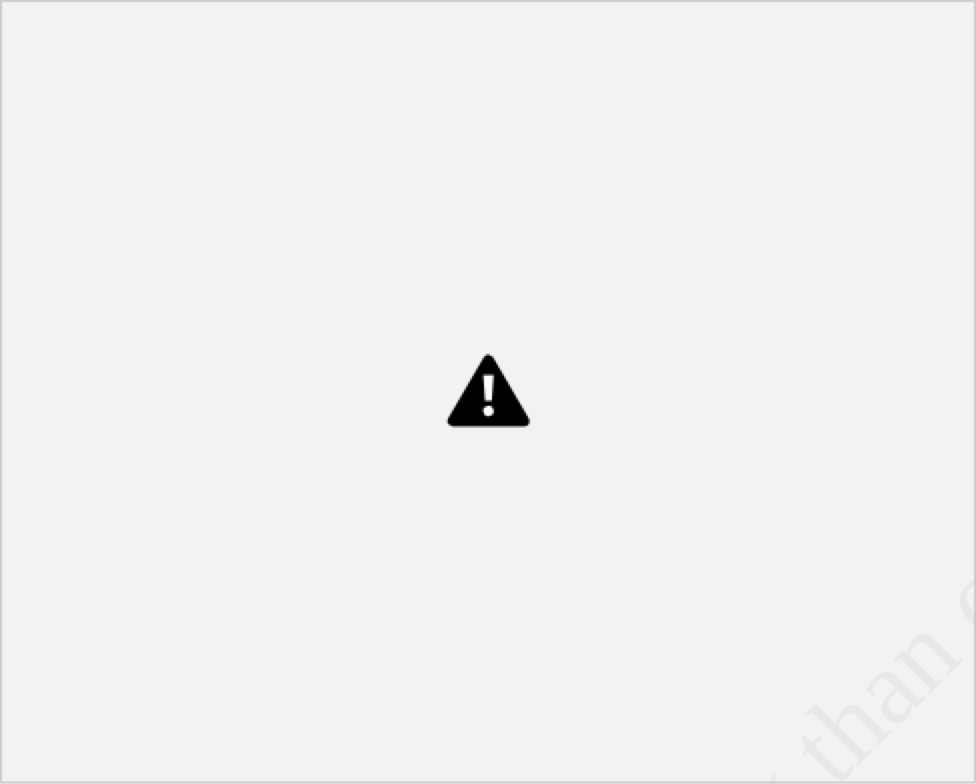

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LAPLACE TRANSFORMS

Inverse Laplace transforms

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Introduction

***Why it is important to understand:* Inverse Laplace transforms**

- **Laplace transforms and their inverses are a mathematical technique which allows us to solve differential equations, by primarily using algebraic methods. This simplification in the solving of equations, coupled with the ability to directly implement electrical components in their transformed form, makes the use of Laplace transforms widespread in both electrical engineering and control systems engineering.**

Introduction

Why it is important to understand: **Inverse Laplace transforms**

- **Laplace transforms have many further applications in mathematics, physics, optics, signal processing, and probability.**
- **This chapter specifically explains how the inverse Laplace transformis determined, which can also involve the use**

of partial fractions. In addition, poles and zeros of transfer functions are briefly explained; these are of importance in stability and control systems.

At the end of this chapter, you should be able to:

- define the inverse Laplace transform
- use a standard list to determine the inverse Laplace transforms of simple functions
- determine inverse Laplace transforms using partial

fractions

- define a pole and a zero
- determine poles and zeros for transfer functions, showing them on a pole–zero diagram

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69.1 Definition of the inverse Laplace transform

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69.2 Inverse Laplace transforms of simple functions

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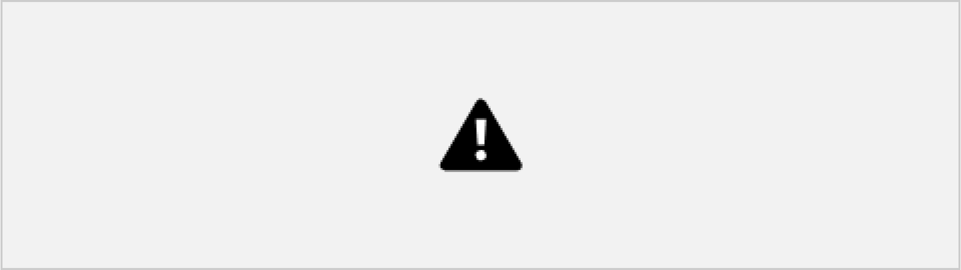
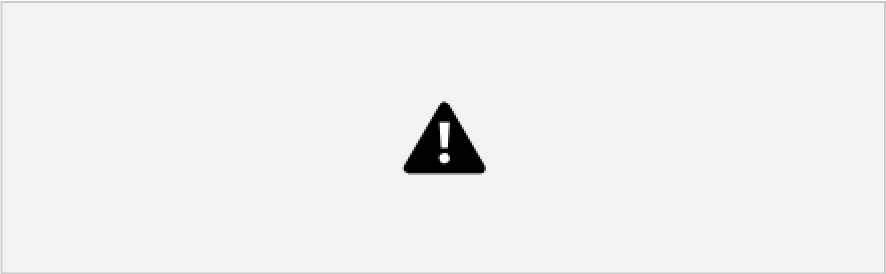


69.2 Inverse Laplace transforms of simple functions



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69.2 Inverse Laplace transforms of simple

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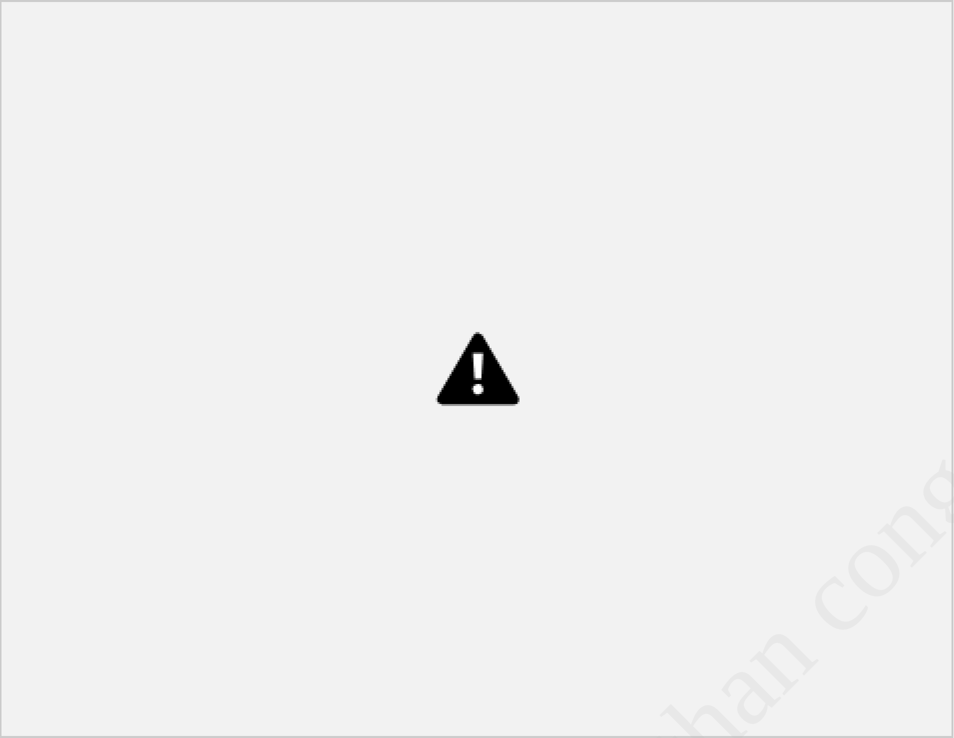

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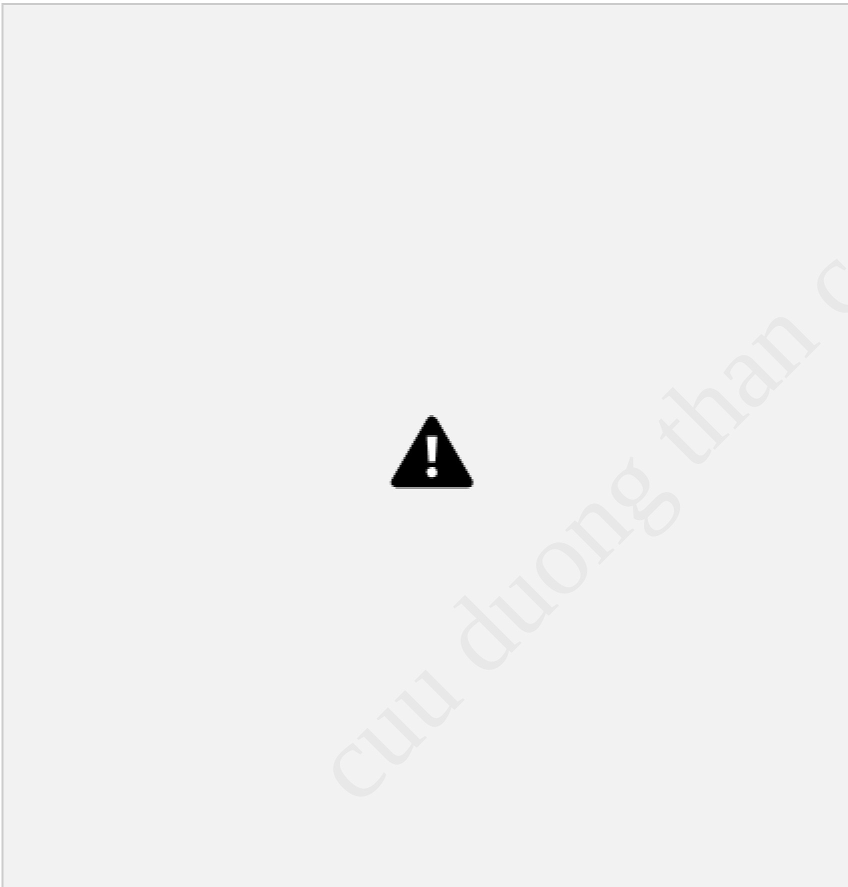

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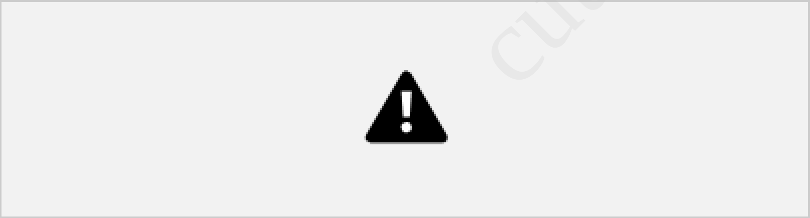

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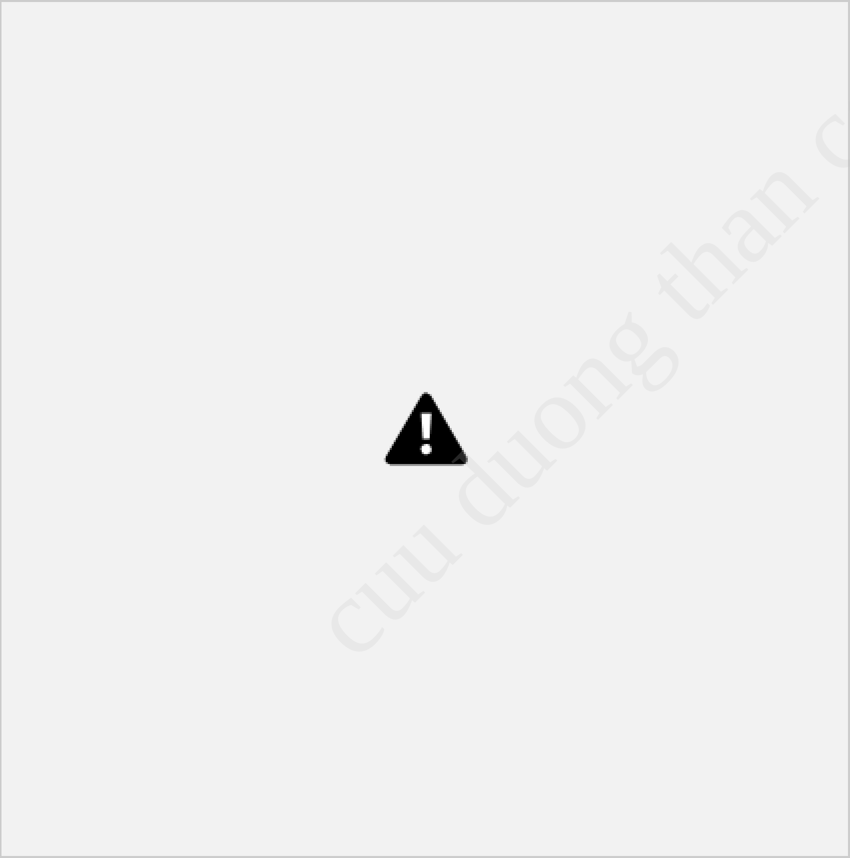

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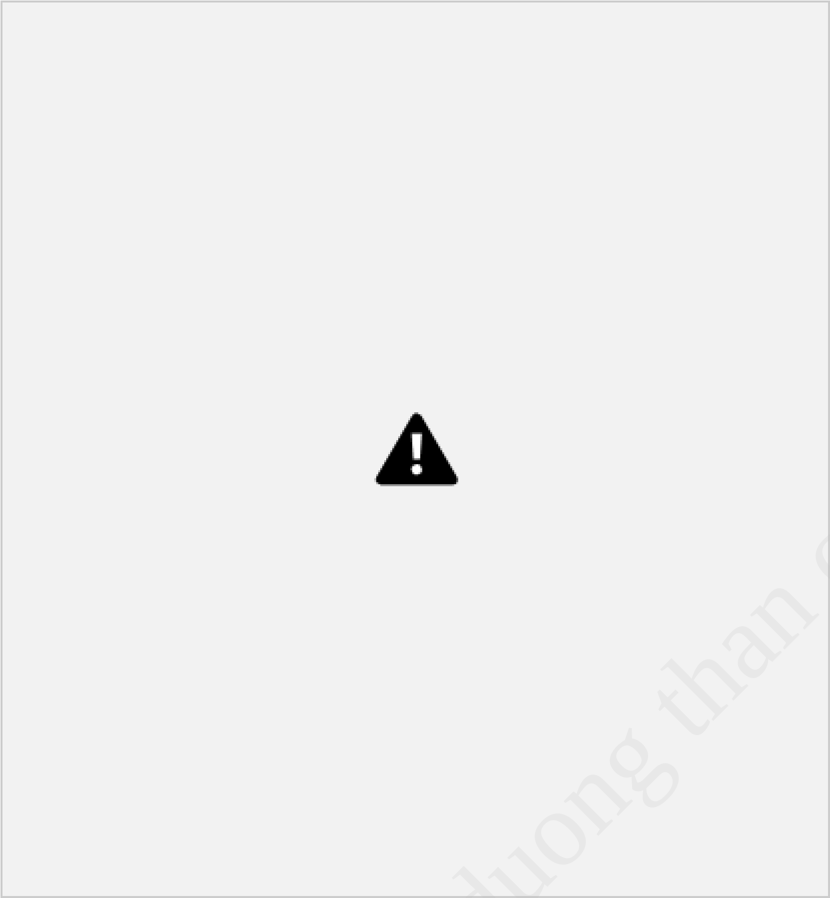

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69.3 Inverse Laplace transforms using partial fractions

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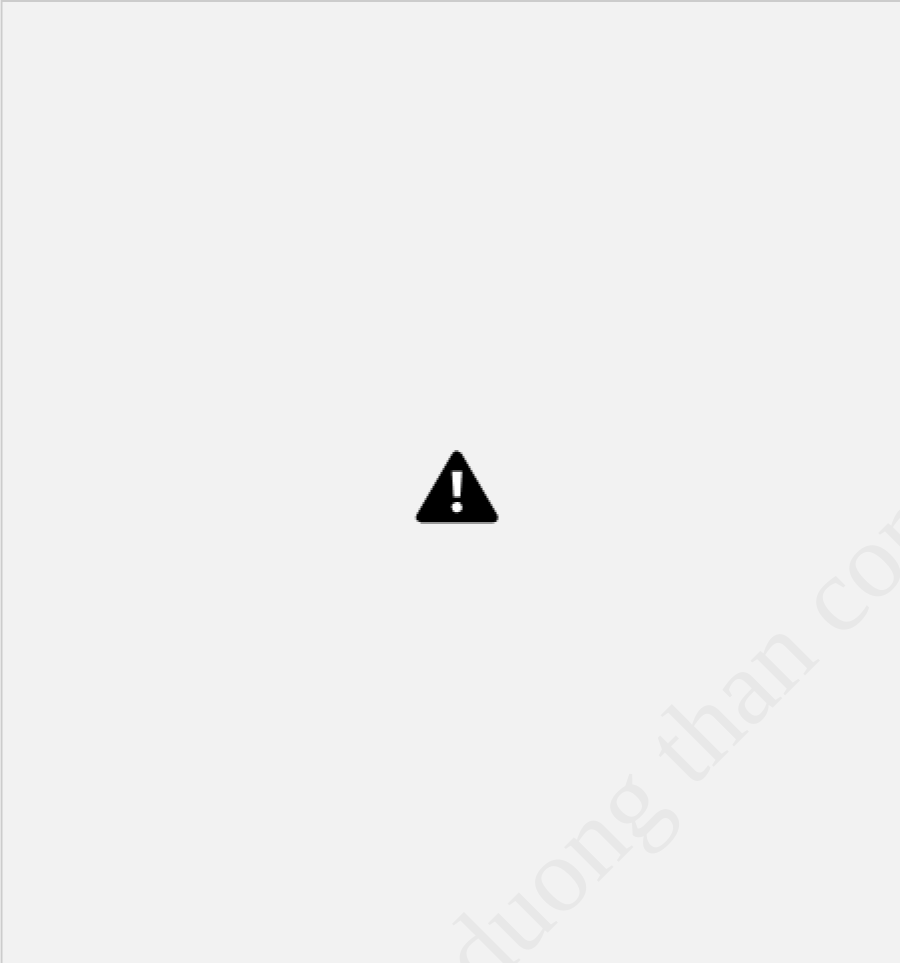

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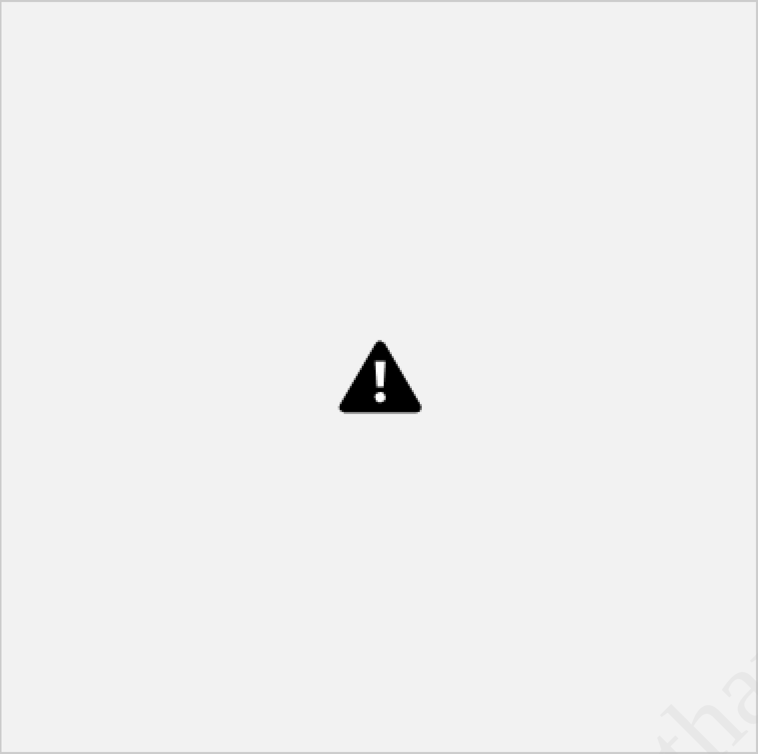

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69.3 Inverse Laplace transforms using partial fractions

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LAPLACE TRANSFORMS

The Laplace transform of the Heaviside function

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Introduction

Why it is important to understand: **The Laplace transform of the Heaviside function**

- **The Heaviside unit step function is used in the** mathematics of control theory and signal processing to represent a signal that switches on at a specified time and stays switched on indefinitely. It is also used in structural mechanics to describe different types of structural loads.

- The Heaviside function has applications in engineering where periodic functions are represented. In many physical situations things change suddenly; brakes are applied, a switch is thrown, collisions occur. The Heaviside unit function is very useful for representing sudden change.

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At the end of this chapter, you should be able to:

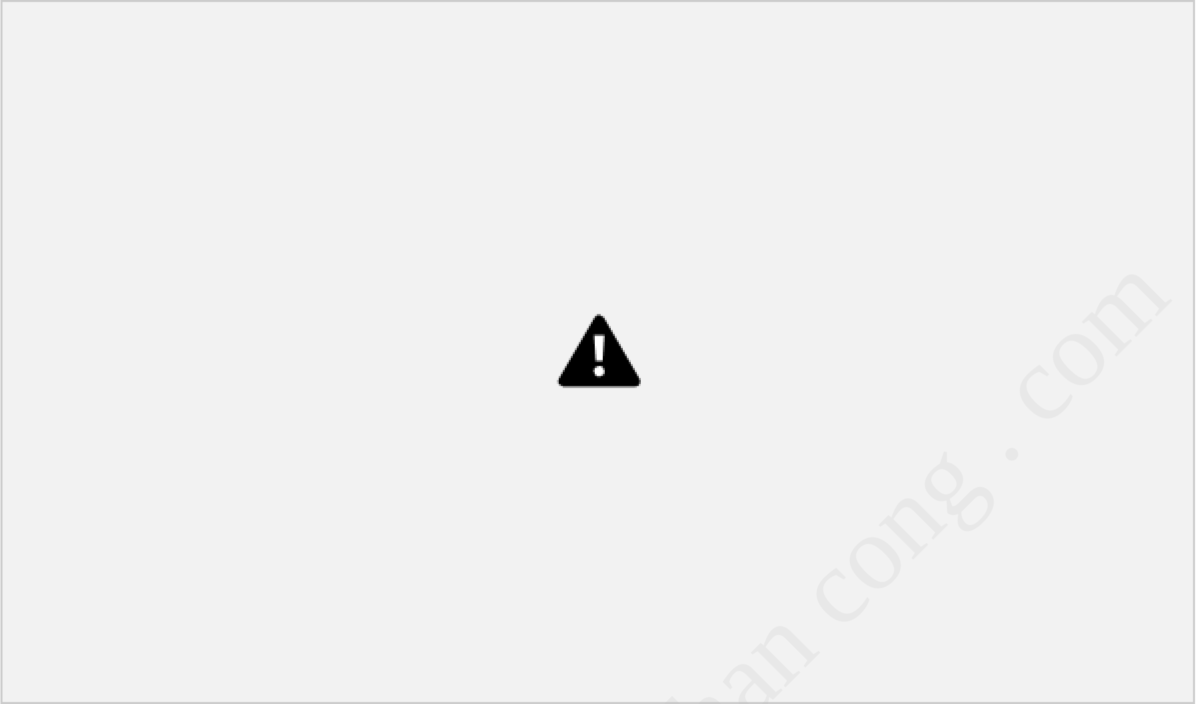
- define the Heaviside unit step function

- use a standard list to determine the Laplace transform of $H(t-c)$

- use a standard list to determine the Laplace transform of $H(t-c) \cdot f(t-c)$

- determine the inverse transforms of Heaviside functions

70.1 Heaviside unit step function



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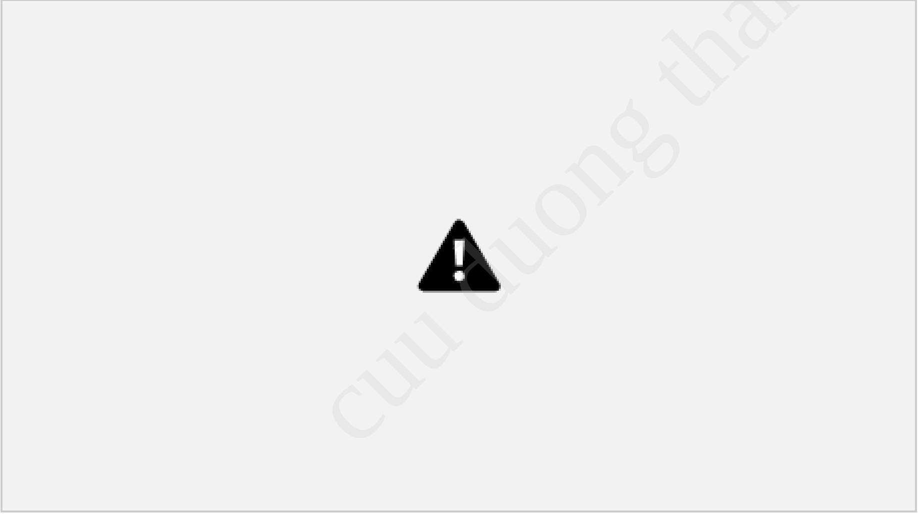

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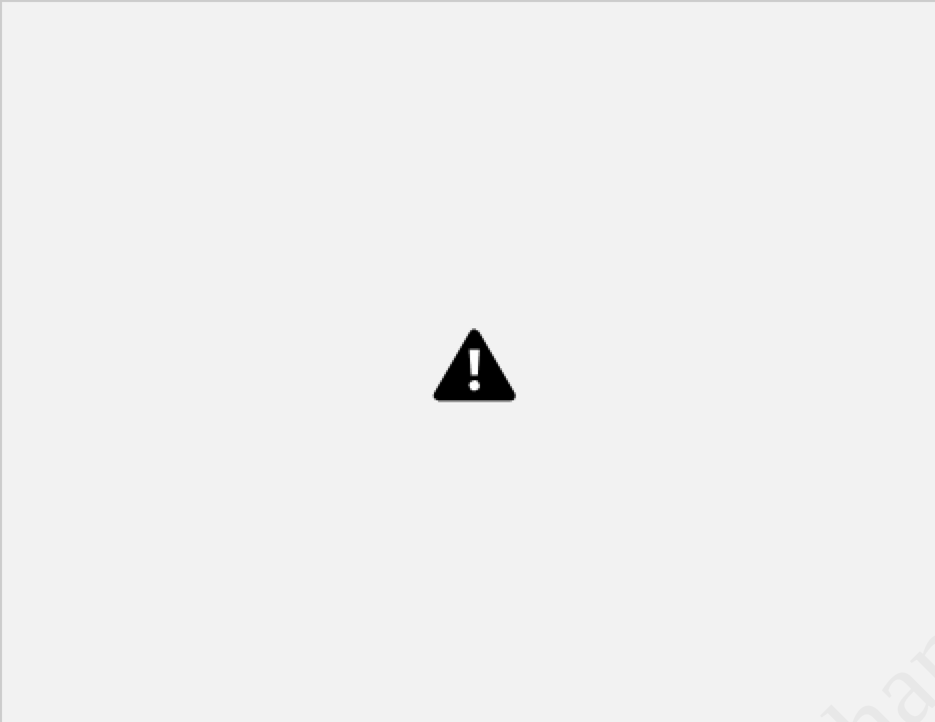

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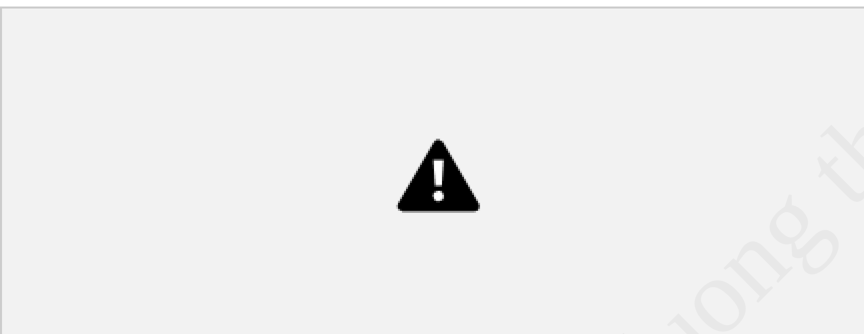

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70.1 Heaviside unit step function

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70.2 Laplace transform of *H(t−c)*

70.3 Laplace transform of $H(t-c) \cdot f(t-c)$

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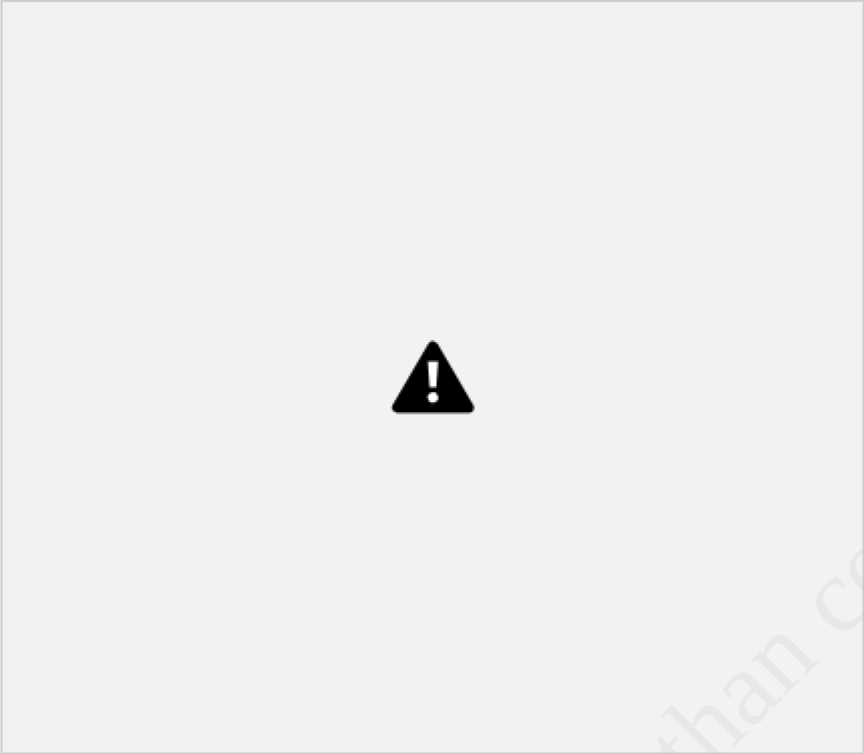

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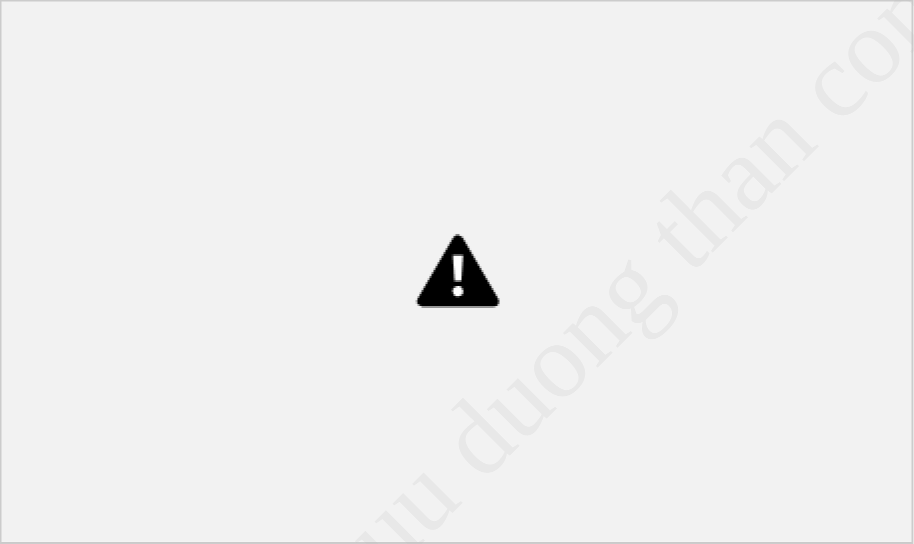
70.4 Inverse Laplace transforms of Heaviside functions



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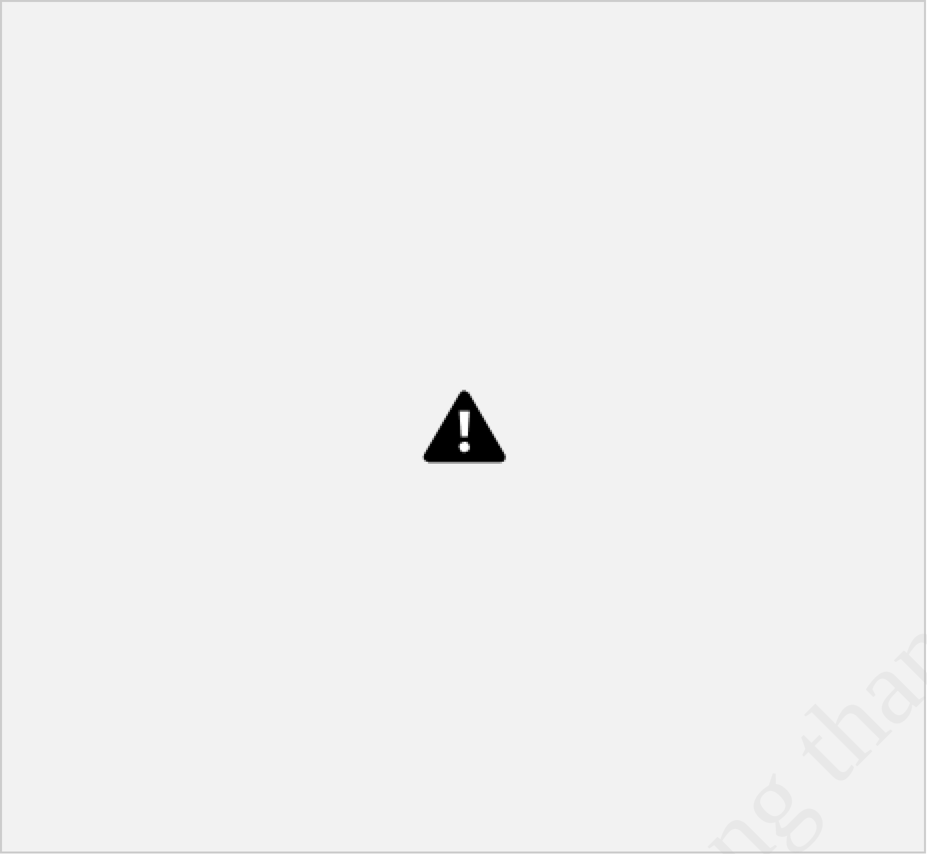

70.4 Inverse Laplace transforms of Heaviside functions

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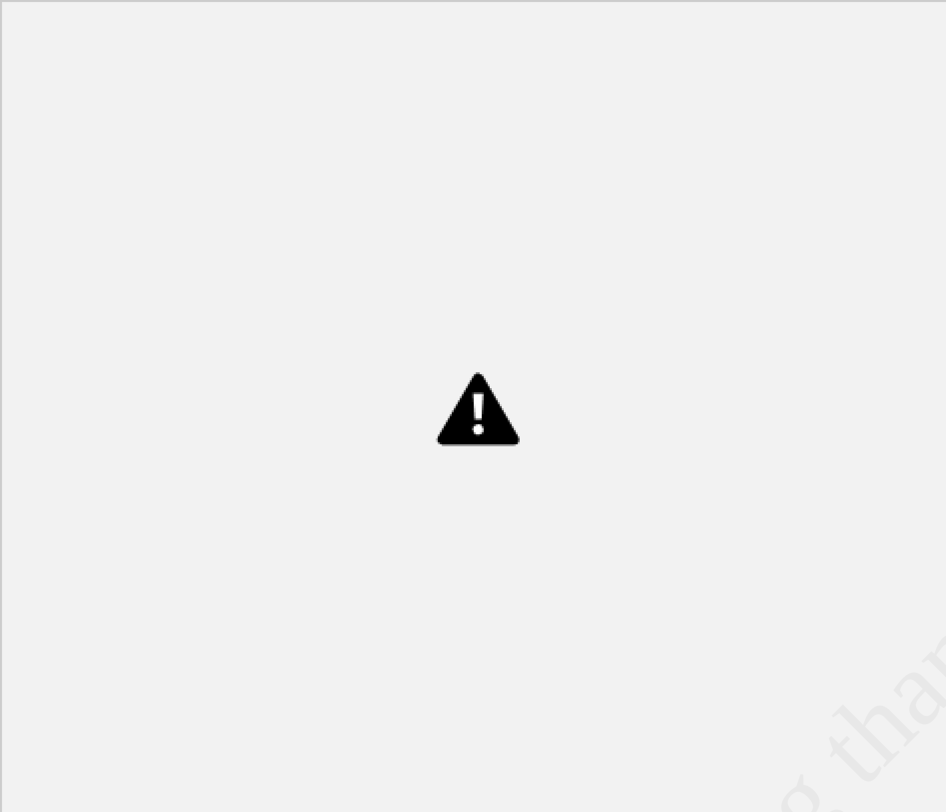

70.4 Inverse Laplace transforms of

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LAPLACE TRANSFORMS
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