Chapter 1:

Vector and Field

1.1 Vector Algebra

a) Vectors ($\vec{A}$) vs. Scalars (A):

- Vector: Magnitude and direction, Ex: Velocity, Force
- Scalar: Magnitude only, Ex: Mass, Charge

b) Unit Vectors

- have magnitude unity denoted by symbol $\vec{a}$ with subscript .

$$\left\{\vec{a}_1;\vec{a}_2;\vec{a}_3\right\}$$

- Useful for expressing vectors in terms of their components

$$\vec{E} = \vec{E}_x + \vec{E}_y + \vec{E}_z = E_x(x,y,z,t)\vec{a}_x + E_y(x,y,z,t)\vec{a}_y + E_z(x,y,z,t)\vec{a}_z$$

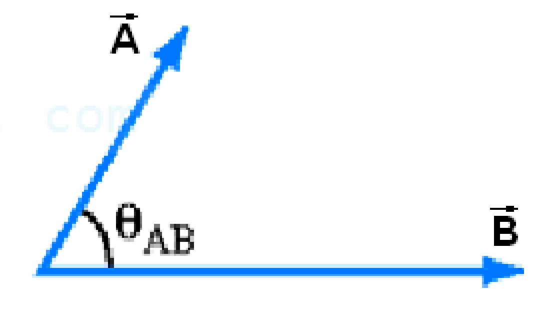
$$\vec{a}_A = \frac{\vec{A}}{A} = \frac{A_1\vec{a}_1 + A_2\vec{a}_2 + A_3\vec{a}_3}{\sqrt{A_1^2 + A_2^2 + A_3^2}}$$

c) Dot Product:

- **is a scalar**

$$\vec{A}.\vec{B} = A_1B_1 + A_2B_2 + A_3B_3 = \mathrm{A.B.}\cos\theta_{AB}$$

$$\vec{A}.\vec{A} = \mathrm{A}^2$$



- **Useful for finding angle between two vectors**

$$\theta_{\mathrm{AB}} = \cos^{-1}\left((\vec{\mathrm{A}}.\vec{\mathrm{B}}) \big/ (\mathrm{A.B}) \right)$$

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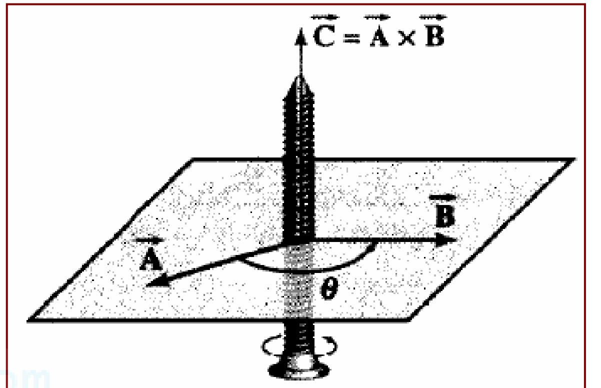
d) Cross Product:

- is a vector, is perpendicular to both $\vec{A}$ and $\vec{B}$

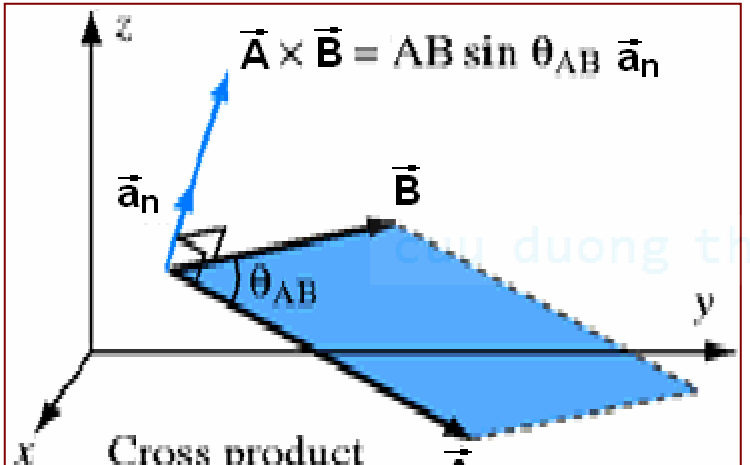
$$\vec{A} \times \vec{A} = 0$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = -\vec{B} \times \vec{A}$$



Right-handed screw rule

- Useful for finding unit vector perpendicular to two vectors.



Cross product

$$\vec{a}_n = \frac{\vec{A} \times \vec{B}}{\left| \vec{A} \times \vec{B} \right|}$$

e) Triple Cross Product:

- is a vector : $$\vec{A}\times(\vec{B}\times\vec{C})$$

- in general : $$\vec{A}\times(\vec{B}\times\vec{C}) \neq \vec{B}\times(\vec{C}\times\vec{A}) \neq \vec{C}\times(\vec{A}\times\vec{B})$$

f) Scalar Triple Product:

- **is a scalar :**

$$\vec{A}.(\vec{B}\times\vec{C}) = \vec{B}.(\vec{C}\times\vec{A}) = \vec{C}.(\vec{A}\times\vec{B})$$

$$= \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

❖ Examples: D1.2

D1.2: Given:

$$\vec{A} = 3\vec{a}_1 + 2\vec{a}_2 + \vec{a}_3$$

$$\vec{B} = \vec{a}_1 + \vec{a}_2 - \vec{a}_3$$

$$\vec{C} = \vec{a}_1 + 2\vec{a}_2 + 3\vec{a}_3$$

a) Compute $\vec{A} + \vec{B} - 4\vec{C}$?

$$= (3+1-4)\vec{a}_1 + (2+1-8)\vec{a}_2 + (1-1-12)\vec{a}_3$$

$$= -5\vec{a}_2 - 12\vec{a}_3$$

$$\Rightarrow \left|\vec{A} + \vec{B} - 4\vec{C}\right| = \sqrt{25 + 144} = 13$$

❖ Examples: D1.2

b) Compute $\vec{A} + 2\vec{B} - \vec{C}$?

$$= (3+2-1)\vec{a}_1 + (2+2-2)\vec{a}_2 + (1-2+3)\vec{a}_3$$

$$= 4\vec{a}_1 + 2\vec{a}_2 - 4\vec{a}_3$$

The unit vector: $$= \frac{4\vec{a}_1 + 2\vec{a}_2 - 4\vec{a}_3}{\left|4\vec{a}_1 + 2\vec{a}_2 - 4\vec{a}_3\right|}$$

$$= \frac{1}{3}\left(2\vec{a}_1 + \vec{a}_2 - 2\vec{a}_3\right)$$

❖ Examples: D1.2

D1.2: Given: $\vec{A} = 3\vec{a}_1 + 2\vec{a}_2 + \vec{a}_3$

$$\vec{B} = \vec{a}_1 + \vec{a}_2 - \vec{a}_3$$

$$\vec{C} = \vec{a}_1 + 2\vec{a}_2 + 3\vec{a}_3$$

c) Compute $\vec{A}.\vec{C}$? $= (3*1) + (2*2) + (1*3) = 10$

d) Compute $\vec{B} \times \vec{C}$?

$$= \begin{vmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \\ 1 & 1 & -1 \\ 1 & 2 & 3 \end{vmatrix} = 5\vec{a}_1 - 4\vec{a}_2 + \vec{a}_3$$

❖ Examples: D1.2

D1.2: Given:

$$\vec{A} = 3\vec{a}_1 + 2\vec{a}_2 + \vec{a}_3$$

$$\vec{B} = \vec{a}_1 + \vec{a}_2 - \vec{a}_3$$

$$\vec{C} = \vec{a}_1 + 2\vec{a}_2 + 3\vec{a}_3$$

e) Compute $\vec{A}.(\vec{B} \times \vec{C})$?

$$= \begin{vmatrix} 3 & 2 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & 3 \end{vmatrix} = 3(3+2) + 2(-1-3) + 1(2-1) = 8$$