1.2: The coordinate systems

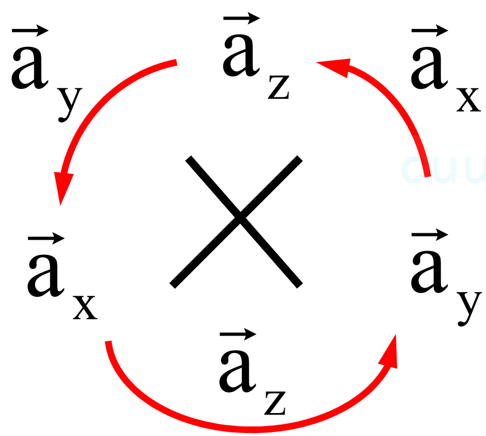
1.2.1 Cartesian Coordinate System:

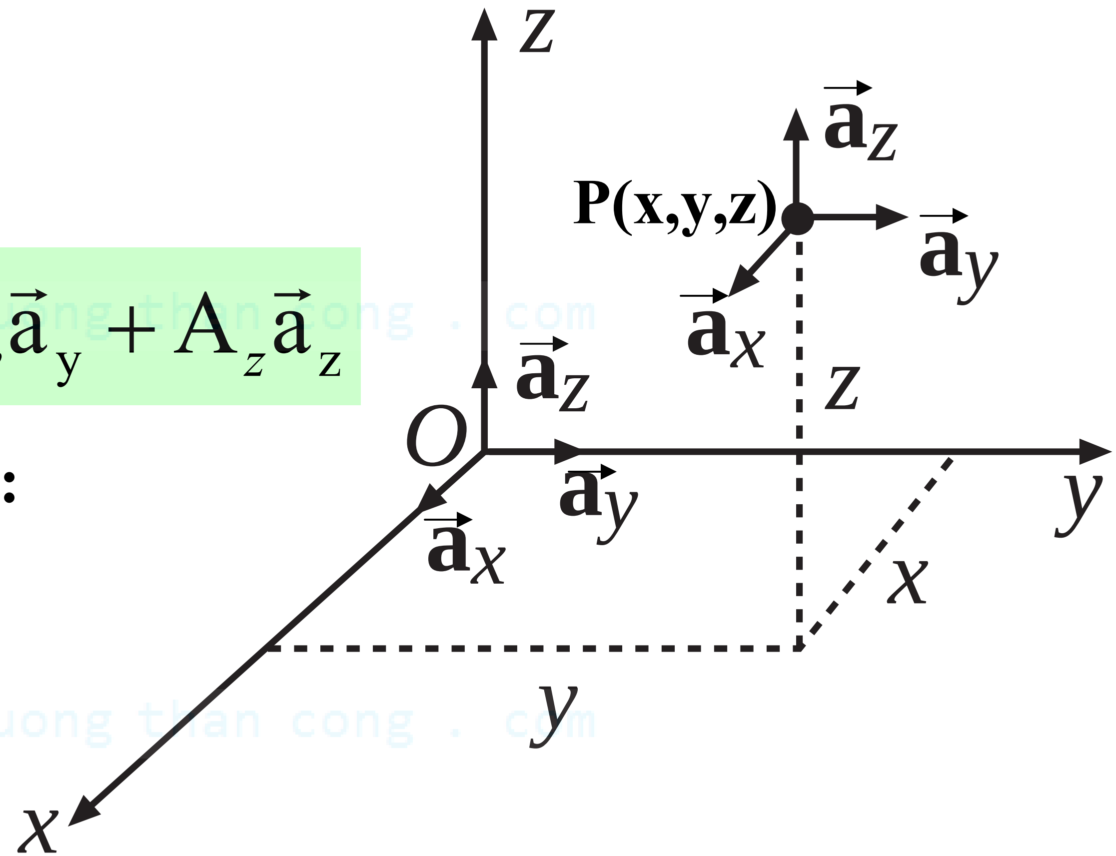
a) Unit Vectors:

* $P(x, y, z)$
* $\{\vec{a}_x, \vec{a}_y, \vec{a}_z\}$

⇨ $$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

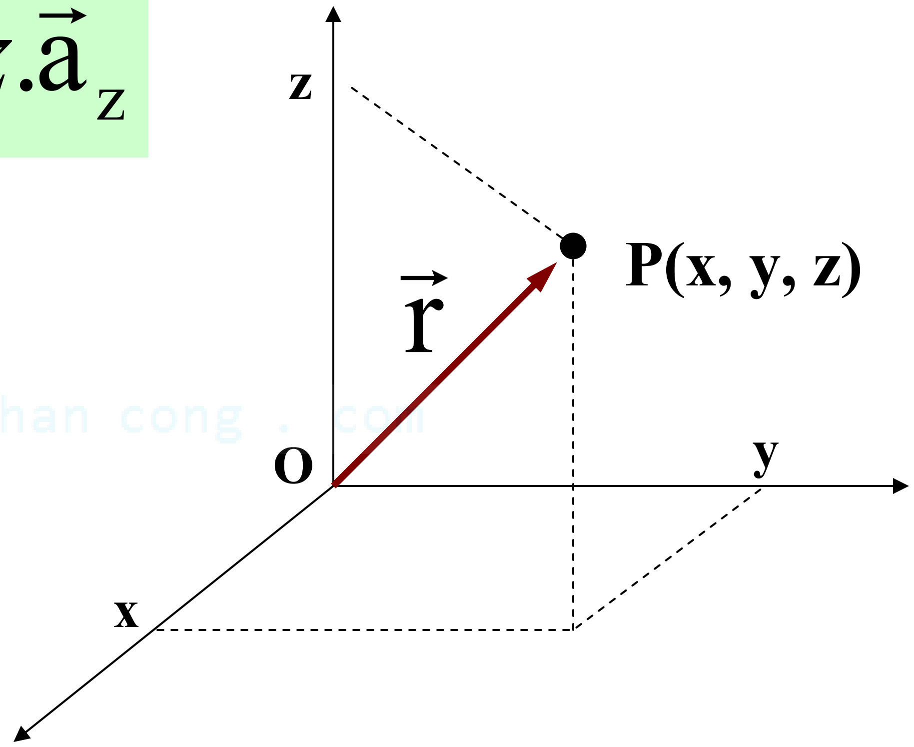
* **Right-handed system :**



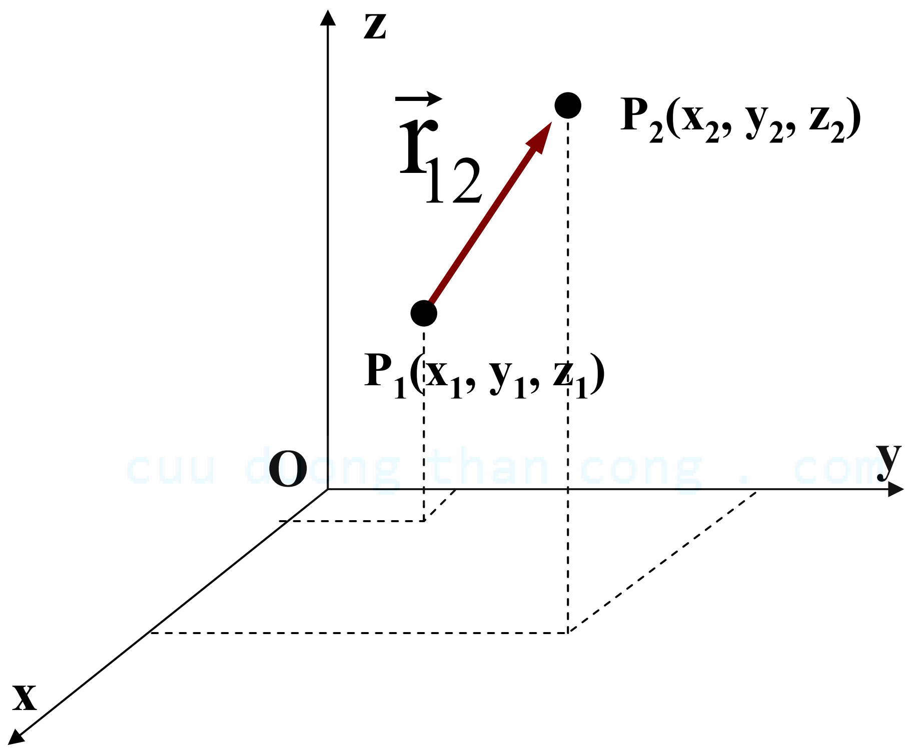


b) Position vector:

$$\vec{r} = x.\vec{a}_x + y.\vec{a}_y + z.\vec{a}_z$$



c) Vector from $P_1(x_1,y_1,z_1)$ to $P_2(x_2,y_2,z_2)$:



$$\vec{r}_{12} = (x_2 - x_1)\vec{a}_x + (y_2 - y_1)\vec{a}_y + (z_2 - z_1)\vec{a}_z$$

❖ Examples: P1.8

Given : $A(12, 0, 0)$, $B(0, 15, 0)$, $C(0, 0, -20)$.

a) Distance from *B* to *C* ?

$$= \left|(0-0)\vec{a}_x + (0-15)\vec{a}_y + (-20-0)\vec{a}_z\right| = 25$$

b) Component of vector from *A* to *C* along vector from *B* to C ?

- **Vector from *A* to C:** $\vec{r}_{AC} = -12\vec{a}_x - 20\vec{a}_z$
- **Unit vector from *B* to C:** $\vec{a}_{BC} = \dfrac{-15\vec{a}_y - 20\vec{a}_z}{25} = \dfrac{-3\vec{a}_y - 4\vec{a}_z}{5}$

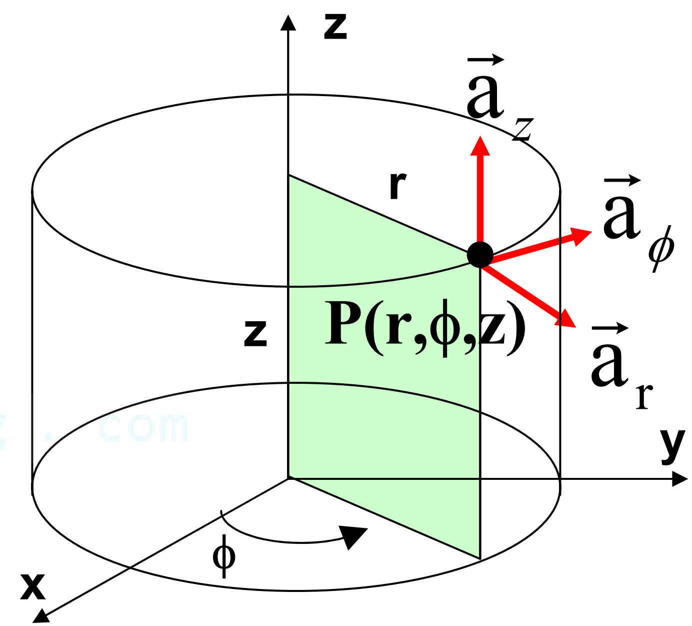
$$\Rightarrow (\vec{r}_{AC}.\vec{a}_{BC})\vec{a}_{BC} = -9.6\vec{a}_y - 12.8\vec{a}_z$$

1.2.2 Cylindrical Coordinate Systems:

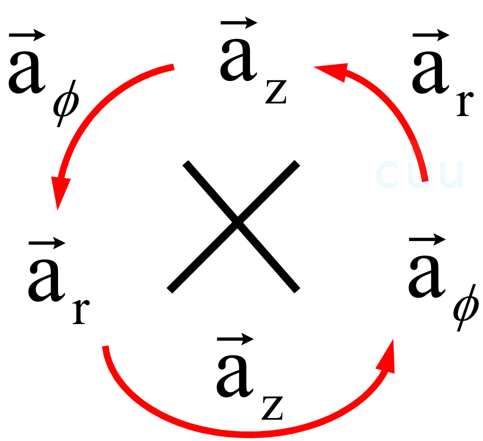
* $P(r, \phi, z)$

* $\{\vec{a}_r, \vec{a}_\phi, \vec{a}_z\}$

$$\Rightarrow \vec{A} = A_r\vec{a}_r + A_\phi\vec{a}_\phi + A_z\vec{a}_z$$



* **Right-handed system :**

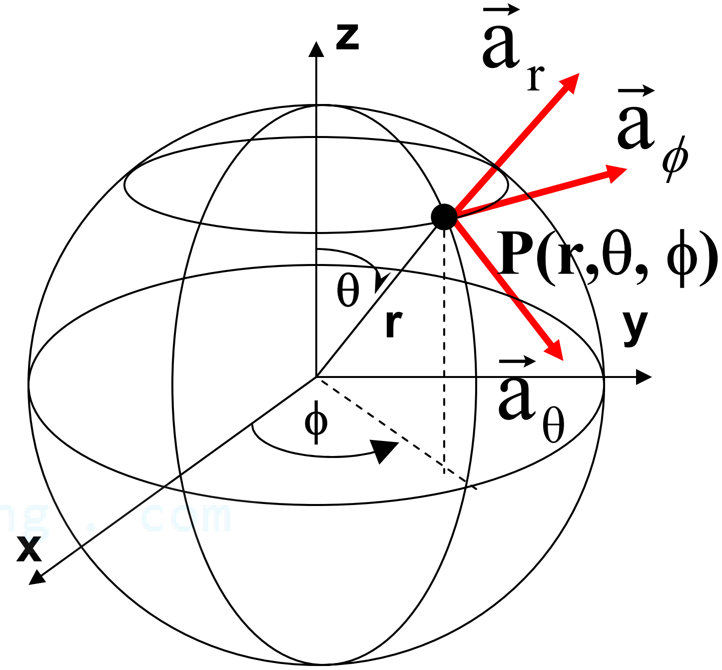


$\vec{a}_\phi \leftarrow \vec{a}_z \leftarrow \vec{a}_r$; $\vec{a}_r \times \vec{a}_\phi = \vec{a}_z$

1.2.3 Spherical Coordinate Systems:

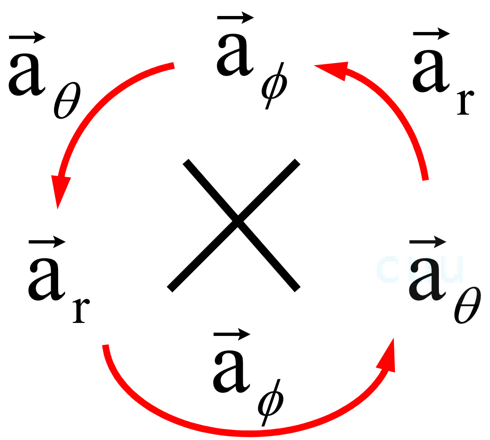
* $P(r, \theta, \phi)$

* $\left\{\vec{a}_r, \ \vec{a}_\theta, \ \vec{a}_\phi\right\}$



⇨ $$\vec{A} = A_r\vec{a}_r + A_\theta\vec{a}_\theta + A_\phi\vec{a}_\phi$$

* **Right-handed system :**



❖**Note:**

- **Cylindrical:** $\left\{\vec{a}_{rc}\right\}$
- **Spherical:** $\left\{\vec{a}_{rs}\right\}$

1.2.4 Conversion of points between co. systems:

Cartesian → **Cylindrical**

$$(x, y, z) \rightarrow \begin{cases} r = \sqrt{x^2 + y^2} \\ \phi = tg^{-1}\dfrac{y}{x} \\ z = z \end{cases}$$

$$\left.\begin{matrix} x = r\cos\phi \\ y = r\sin\phi \\ z = z \end{matrix}\right\} \leftarrow (r, \phi, z)$$

1.2.4 Conversion of points between co. systems:

Cartesian → **Spherical**

$$(x, y, z) \rightarrow \begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = tg^{-1} \dfrac{\sqrt{x^2 + y^2}}{z} \\ \phi = tg^{-1} \dfrac{y}{x} \end{cases}$$

$$\left. \begin{aligned} x &= r \sin\theta \cos\phi \\ y &= r \sin\theta \sin\phi \\ z &= r \cos\theta \end{aligned} \right\} \leftarrow (r, \theta, \phi)$$

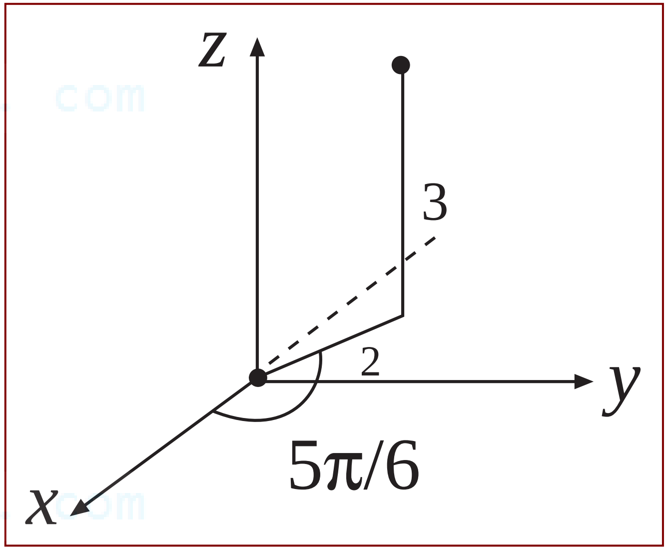
❖ Example: Determine x, y, z ?

Note:	x	=	$r\cos\phi$	x	=	$r\sin\theta\cos\phi$
	y	=	$r\sin\phi$	y	=	$r\sin\theta\sin\phi$
	z	=	z	z	=	$r\cos\theta$

D1.7 (a) **(2, 5π/6, 3) in cylindrical coordinates**

$$\left.\begin{aligned} x &= 2\cos 5\pi/6 = -\sqrt{3} \\ y &= 2\sin 5\pi/6 = 1 \end{aligned}\right\} \sqrt{3+1} = 2$$

$$z = 3$$



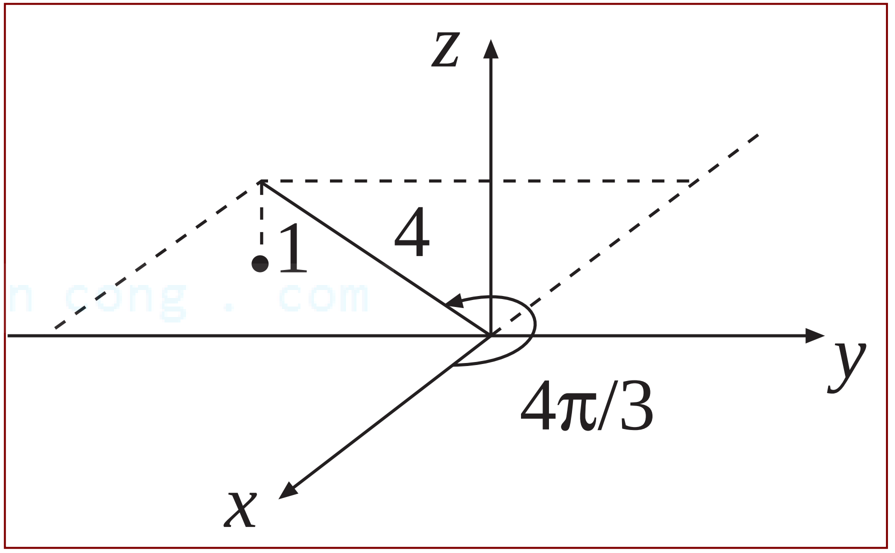
❖Example: Determine x, y, z ?

Note:	x	=	$r\cos\phi$	x	=	$r\sin\theta\cos\phi$
	y	=	$r\sin\phi$	y	=	$r\sin\theta\sin\phi$
	z	=	z	z	=	$r\cos\theta$

D1.7 (b) $(4,\ 4\pi/3,\ -1)$ in cylindrical coordinates ?

$$\left.\begin{aligned} x &= 4\cos 4\pi/3 = -2 \\ y &= 4\sin 4\pi/3 = -2\sqrt{3} \end{aligned}\right\} \sqrt{4+12} = 4$$

$$z = -1$$

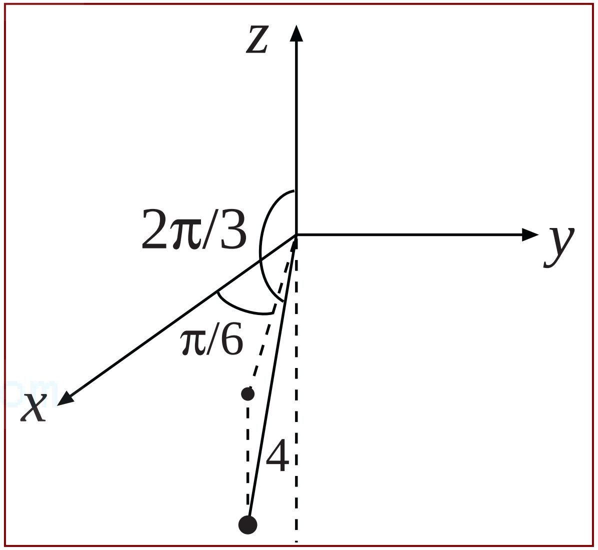


❖ Example: Determine x, y, z ?

Note:	x	=	$r\cos\phi$	x	=	$r\sin\theta\cos\phi$
	y	=	$r\sin\phi$	y	=	$r\sin\theta\sin\phi$
	z	=	z	z	=	$r\cos\theta$

D1.7 **(c)** $(4, 2\pi/3, \pi/6)$ in spherical coordinates ?

$$\left.\begin{aligned} x &= 4\sin\frac{2\pi}{3}\cos\frac{\pi}{6} = 3 \\ y &= 4\sin\frac{2\pi}{3}\sin\frac{\pi}{6} = \sqrt{3} \\ z &= 4\cos\frac{2\pi}{3} = -2 \end{aligned}\right\} \sqrt{9+3+4} = 4$$



❖Example: Determine x, y, z ?

Note:

x	=	$r\cos\phi$	x	=	$r\sin\theta\cos\phi$
y	=	$r\sin\phi$	y	=	$r\sin\theta\sin\phi$
z	=	z	z	=	$r\cos\theta$

D1.7 **(d)** $\left(\sqrt{8}, \pi/4, \pi/3\right)$ in spherical coordinates ?

$$\left.\begin{aligned} x &= \sqrt{8}\sin\frac{\pi}{4}\cos\frac{\pi}{3} = 1 \\ y &= \sqrt{8}\sin\frac{\pi}{4}\sin\frac{\pi}{3} = \sqrt{3} \\ z &= \sqrt{8}\cos\frac{\pi}{4} = 2 \end{aligned}\right\} \sqrt{1+3+4} = \sqrt{8}$$