# 1.8 Boundary Conditions For EM:



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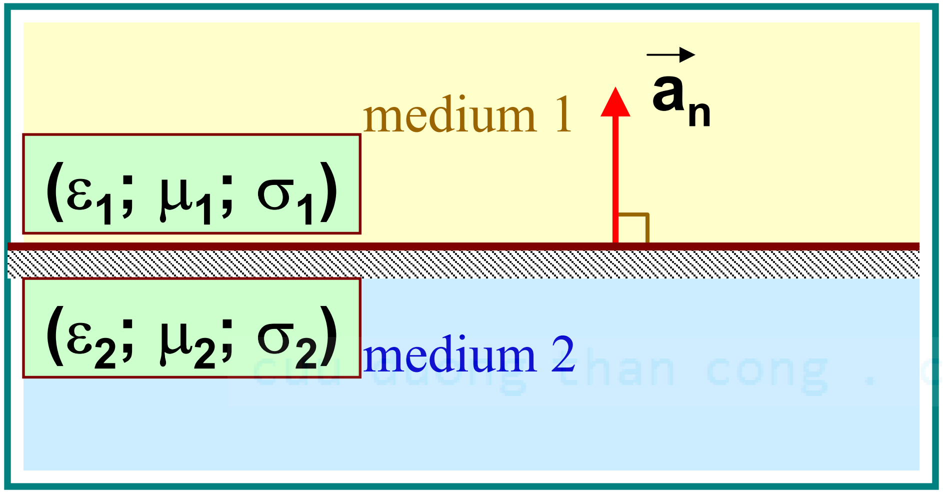
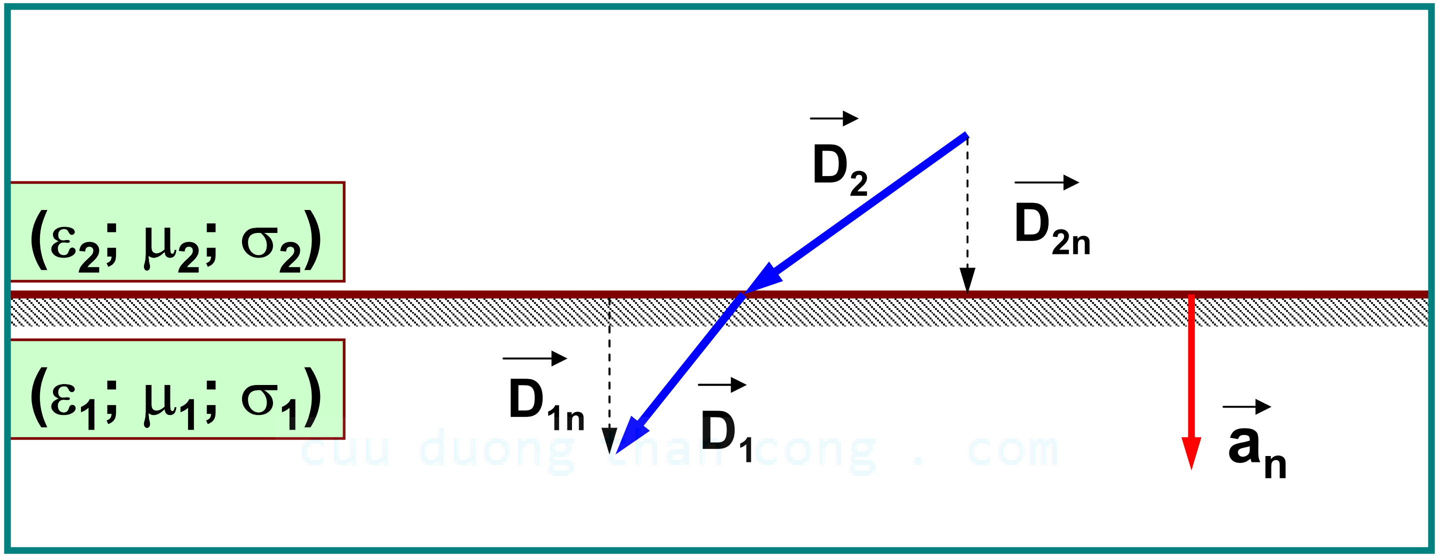
# a) Introduction:

- The conditions that the field must satisfy at the interface .




- Attention : $\vec{a}_n : 2 \rightarrow 1$

# b) The conditions on the normal components:




$$D_{1n} - D_{2n} = \rho_s$$
$$B_{1n} - B_{2n} = 0$$
$$J_{1n} - J_{2n} = -\frac{\partial \rho_s}{\partial t}$$

$$\vec{a}_n(\vec{D}_1 - \vec{D}_2) = \rho_s$$
$$\vec{a}_n(\vec{B}_1 - \vec{B}_2) = 0$$
$$\vec{a}_n(\vec{J}_1 - \vec{J}_2) = -\frac{\partial \rho_s}{\partial t}$$

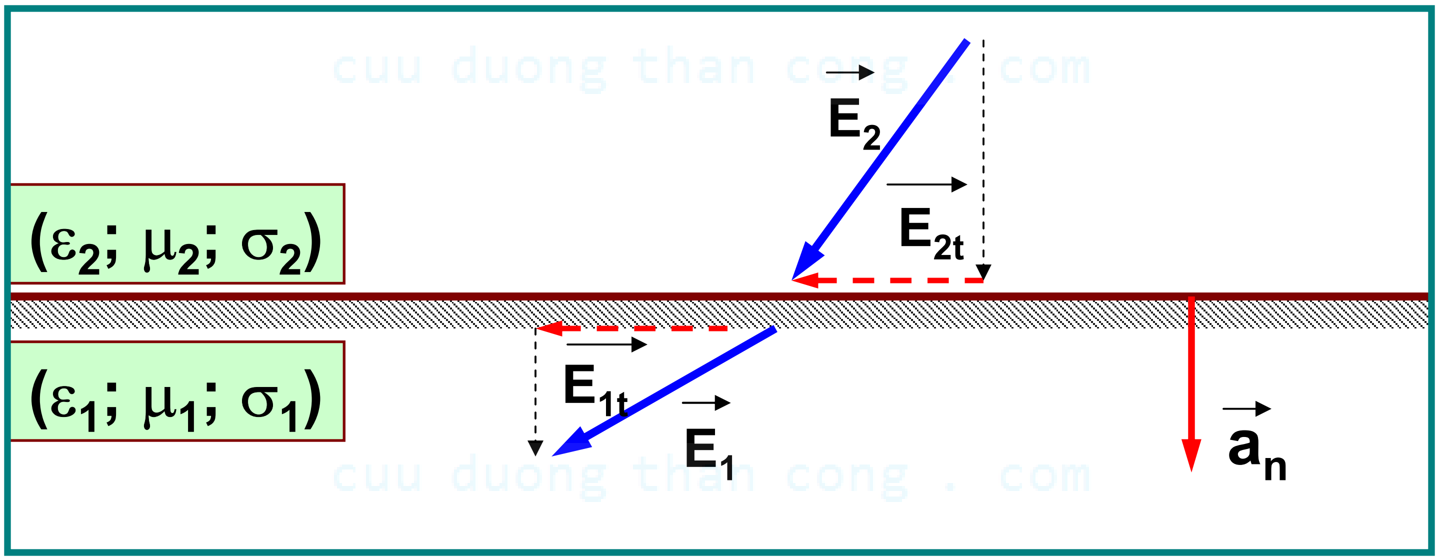
$$(\vec{D}_{1n} - \vec{D}_{2n}) = \rho_s.\vec{a}_n$$
$$(\vec{B}_{1n} - \vec{B}_{2n}) = 0$$
$$(\vec{J}_{1n} - \vec{J}_{2n}) = -\frac{\partial \rho_s}{\partial t}.\vec{a}_n$$

## c) The conditions on the tangential components:

$$H_{1t} - H_{2t} = J_S$$
$$E_{1t} - E_{2t} = 0$$

$$\vec{a}_n \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_S$$
$$\vec{a}_n \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$(\vec{H}_{1t} - \vec{H}_{2t}) = \vec{J}_S \times \vec{a}_n$$
$$(\vec{E}_{1t} - \vec{E}_{2t}) = 0$$



# d) The Special cases :

## ❖ case 1: lossless linear media

No free charges and no surface currents at interface between two lossless media. $\therefore J_s = 0,\ \rho_S = 0$

**Electric Field**

$$\vec{E}_{1t} = \vec{E}_{2t} \Rightarrow \frac{\vec{D}_{1t}}{\vec{D}_{2t}} = \frac{\varepsilon_1}{\varepsilon_2}$$

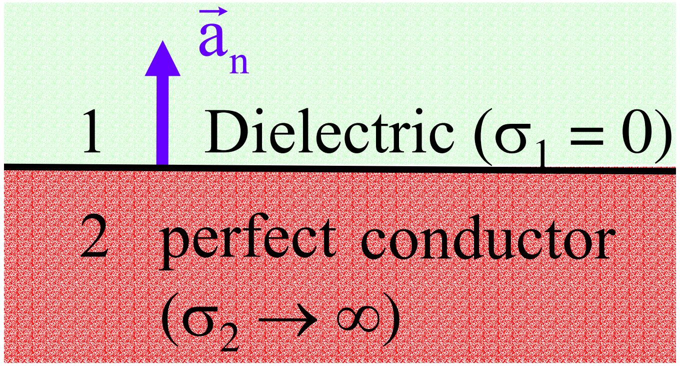
$$\vec{D}_{1n} = \vec{D}_{2n} \Rightarrow \varepsilon_1 \vec{E}_{1n} = \varepsilon_2 \vec{E}_{2n}$$

**Magnetic Field**

$$\vec{H}_{1t} = \vec{H}_{2t} \Rightarrow \frac{\vec{B}_{1t}}{\vec{B}_{2t}} = \frac{\mu_1}{\mu_2}$$

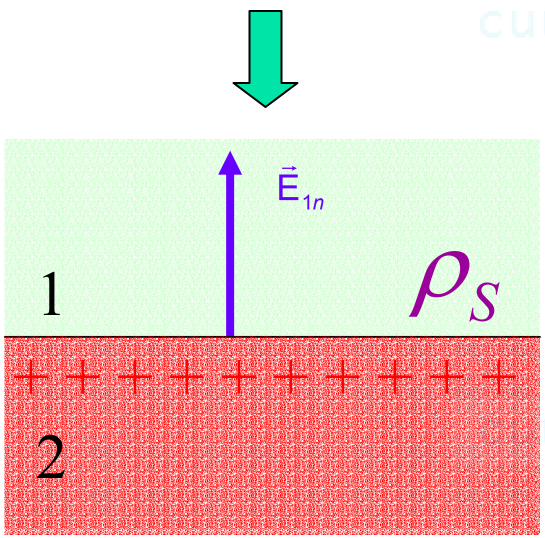
$$\vec{B}_{1n} = \vec{B}_{2n} \Rightarrow \mu_1 \vec{H}_{1n} = \mu_2 \vec{H}_{2n}$$

# ❖ case 2: Perfect Conductor

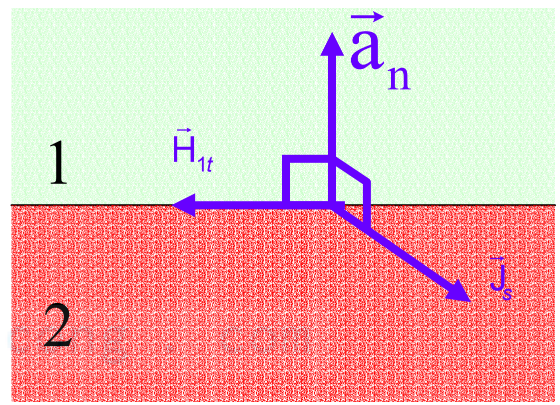



| Medium 1 | Medium 2 |
|---|---|
| $E_{1t} = 0$ | $E_{2t} = 0$ |
| $\vec{a}_n \times \vec{H}_1 = \vec{J}_s$ | $H_{2t} = 0$ |
| $\vec{a}_n \cdot \vec{D}_1 = \rho_S$ | $D_{2n} = 0$ |
| $B_{1n} = 0$ | $B_{2n} = 0$ |



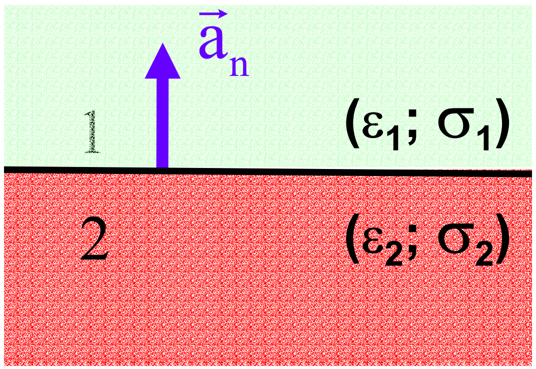


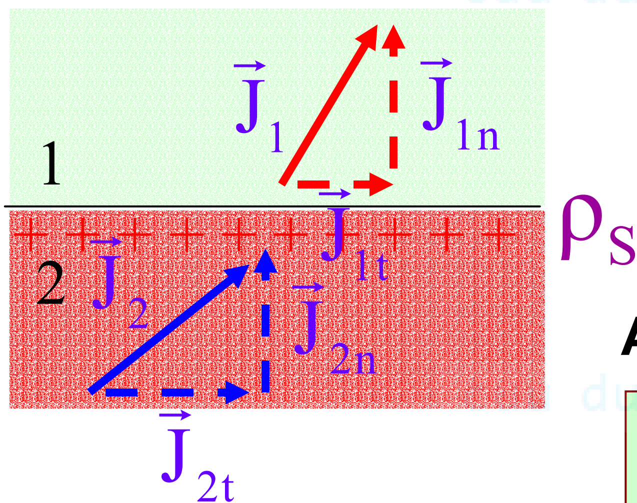

$$E_{1n} = \frac{\rho_S}{\varepsilon_1}$$




$$H_{1t} = J_S$$

# ❖Special case c: Two Conducting media




**Under electrostatic conditions:**

$$\vec{E}_{1t} = \vec{E}_{2t} \Rightarrow \frac{\vec{J}_{1t}}{\sigma_1} = \frac{\vec{J}_{2t}}{\sigma_2}$$

$$\vec{J}_{1n} = \vec{J}_{2n} \Rightarrow \sigma_1 \vec{E}_{1n} = \sigma_2 \vec{E}_{2n}$$

**And at the boundary :**

$$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_S$$