2.8 Steady Electric Currents :

a) The Current Density Vector $\vec{J}$:

- Steady Current is also called DC Current .

- From Continuity equation : $\text{div}\vec{J} = -\dfrac{\partial \rho_V}{\partial t}$

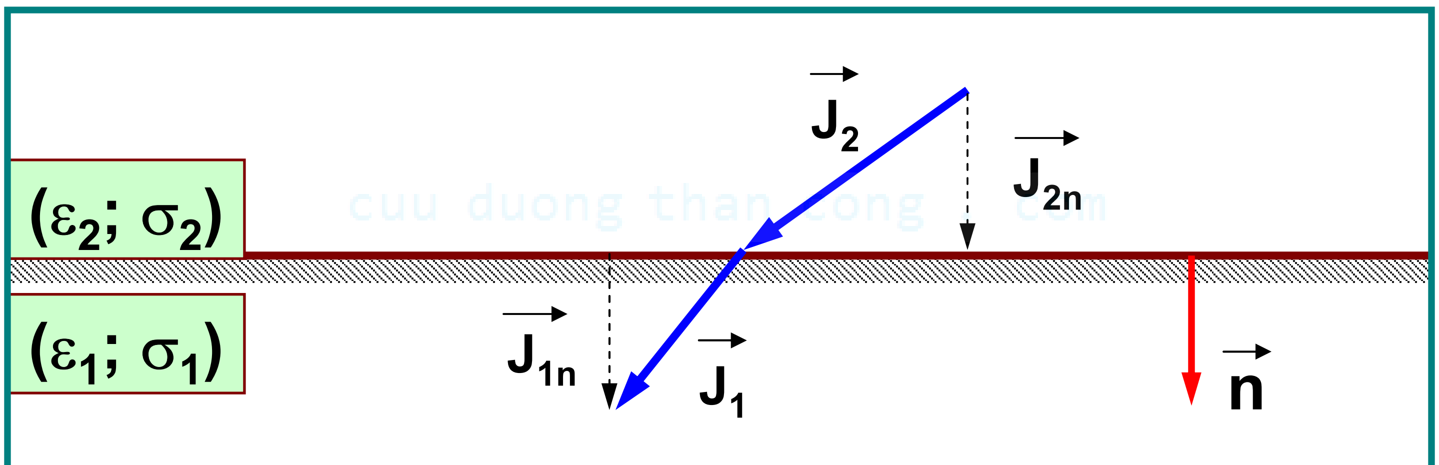
(Electrostatic : $\partial/\partial t = 0$) ⟹ $\text{div}\vec{J} = 0$

- Boundary Condition of Current Density :

$$J_{1n} - J_{2n} = -\frac{\partial \rho_S}{\partial t}$$

$$\vec{a}_n(\vec{J}_1 - \vec{J}_2) = -\frac{\partial \rho_S}{\partial t}$$

$$(\vec{J}_{1n} - \vec{J}_{2n}) = -\frac{\partial \rho_S}{\partial t}.\vec{a}_n$$



b) Electrostatics in Conducting Medium :

- **Field quantities: $\vec{E}$, $\vec{D}$, φ and $\vec{J}$.**

- **Relationship between $\vec{E}$ and $\vec{J}$:** $\vec{J} = \sigma\vec{E}\ [A/m^2]$

- **Potential :** $\mathrm{div}\vec{J} = 0$ ⇒ $\mathrm{div}[\sigma(\mathrm{grad}\varphi)] = 0$

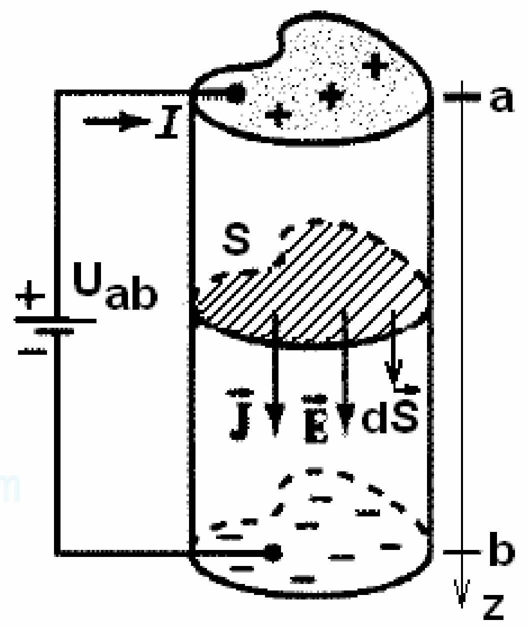
The solution :

- **If σ = const :** → $\Delta\varphi = 0$
- **If $\sigma \neq$ const :** → $\mathrm{div}[\sigma(\mathrm{grad}\varphi)] = 0$

c) Resistance of a Conductor :

❖ **Defined :** $$R = \frac{U_{ab}}{I} \; [\Omega]$$

$$G = \frac{1}{R} = \text{conductance}[\text{S or } \mho]$$



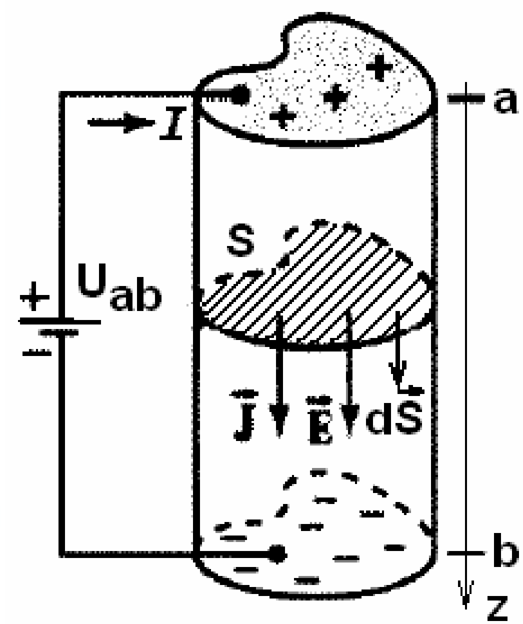
❖ General procedure to Compute Resistance :

i. **Choose coordinate system .**

ii. **Assume U_{ab} = potential difference between conductor terminals .**

iii. **Solve $\Delta\varphi = 0$ or $\mathrm{div}[\sigma(\mathrm{grad}\varphi)] = 0$ to find φ .**

$$\Rightarrow \quad \vec{E} = -\mathrm{grad}(\varphi). \Rightarrow \quad \vec{J} = \sigma\vec{E}.$$



iv. Determine the current: $I = \int_S \vec{J}.d\vec{S}$

v. Obtain $R = \left|\dfrac{U_{ab}}{I}\right| \; (\Omega)$

d) Joule's Law :

- **Current Density** → **Power dissipation as heat**

- **Dissipated Power Density:** $p = \vec{J}.\vec{E} = \sigma E^2 = J^2/\sigma \ [W/m^3]$

- **Total Power Dissipation:**

$$P = \int_V p.dV = \int_V \sigma E^2.dV \ [W]$$

e) Analogy between $\vec{D}$ and $\vec{J}$:

Charge-free Medium	Conducting Medium
$\vec{E}, \varphi, \varepsilon, \vec{D} = \varepsilon\vec{E}, \ldots$ ⟺	$\vec{E}, \varphi, \sigma, \vec{J} = \sigma\vec{E}, \ldots$
$\mathrm{rot}\,\vec{E} = 0 \;\; ; \;\; \vec{E} = -\mathrm{grad}(\varphi)$	$\mathrm{rot}\,\vec{E} = 0 \;\; ; \;\; \vec{E} = -\mathrm{grad}(\varphi)$
$\mathrm{div}\,\vec{D} = 0$	$\mathrm{div}\,\vec{J} = 0$
$E_{1t} - E_{2t} = 0; \;\; D_{1n} - D_{2n} = 0$	$E_{1t} - E_{2t} = 0; \;\; J_{1n} - J_{2n} = 0$

❖ **We can obtain the current density by substituting $\vec{D}$ for $\vec{J}$.**