4.2 Time – Harmonic Fields

cuu duong than cong . com

a) Introduction :

- **Time – harmonic Field varies sinusoidal with time.**

$$\vec{E}(x,y,z,t) = E_{mx}(x,y,z)\cos[\omega t + \psi_x(x,y,z)]\vec{\mathbf{a}}_x$$
$$+ E_{my}(x,y,z)\cos[\omega t + \psi_y(x,y,z)]\vec{\mathbf{a}}_y$$
$$+ E_{mz}(x,y,z)\cos[\omega t + \psi_z(x,y,z)]\vec{\mathbf{a}}_z$$

- **Time – harmonic Field : practical value.**

⟹ **If not, Fourier techniques can be used .**

b) The phasor:

- The phasor is defined: complex function

Time domain: $$\vec{E} = E_{mx}(z)\cos[\omega t + \psi_x(z)]\vec{\mathbf{a}}_x$$

Phasor domain: $$\dot{\vec{E}} = E_{mx}(z).e^{j\psi(z)}\vec{\mathbf{a}}_x = E_{mx}(z)\angle\psi(z).\vec{\mathbf{a}}_x$$

- Relation between the field and the phasor :

$$\dot{\vec{E}}(z) \Leftrightarrow \vec{E}(z,t) = \mathrm{Re}\{\dot{\vec{E}}(z) * e^{j\omega t}\}$$

- Property : $$\frac{\partial\vec{E}(z,t)}{\partial t} \Leftrightarrow j\omega * \dot{\vec{E}}(z)$$

c) Maxwell's equations in phasor form:

❖ **In material media : σ , ε , μ = const , Maxwell's equations :**

$$\operatorname{rot}\vec{H} = \vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t}$$

$$\operatorname{rot}\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$$

$$\operatorname{div}\vec{E} = \rho_V / \varepsilon$$

$$\operatorname{div}\vec{H} = 0$$

⇒

$$\operatorname{rot}\dot{\vec{H}} = (\sigma + j\omega\varepsilon)\dot{\vec{E}}$$

$$\operatorname{rot}\dot{\vec{E}} = -j\omega\mu\,\dot{\vec{H}}$$

$$\operatorname{div}\dot{\vec{E}} = \dot{\rho}_V / \varepsilon$$

$$\operatorname{div}\dot{\vec{H}} = 0$$

❖ **And constitutive relations :**

$$\dot{\vec{J}} = \sigma\dot{\vec{E}} \;;\; \dot{\vec{D}} = \varepsilon\dot{\vec{E}} \;;\; \dot{\vec{B}} = \mu\dot{\vec{H}}$$

❖ Example1: Maxwell's equations in phasor

In a medium characterized by $\sigma = 0$, $\mu = \mu_o$, ε_o, and

$$\vec{\mathbf{E}} = 20 \sin(10^8 t - \beta z)\, \vec{\mathbf{a}}_y \text{ V/m}$$

calculate β and $\vec{\mathbf{H}}$.

- **Method 1:** **Solve directly in time domain (see 1.7) .**
- **Method 2:** **Using phasors :** $\vec{E}(z,t) \Longrightarrow \dot{\vec{E}} = 20.e^{-j\beta z}\vec{a}_y \text{ (V/m)}$

$$\Longrightarrow \text{rot}\dot{\vec{E}} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 20.e^{-j\beta z} & 0 \end{vmatrix} = j20\beta.e^{-j\beta z}\vec{a}_x$$

$$\Longrightarrow \dot{\vec{H}} = -\frac{1}{j\omega\mu_0}\text{rot}\dot{\vec{E}} = -\frac{20\beta}{\omega\mu_0}.e^{-j\beta z}\vec{a}_x$$

❖ Example1: Maxwell's equations in phasor

$$\Rightarrow \text{rot}\dot{\vec{H}} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -20\beta.e^{-j\beta z}/\omega\mu_0 & 0 & 0 \end{vmatrix} = \frac{j20\beta^2}{\omega\mu_0}.e^{-j\beta z}\vec{a}_y$$

Notice that $\text{rot}\dot{\vec{H}} = j\omega\varepsilon_0\dot{\vec{E}} = j\omega\varepsilon_0 20.e^{-j\beta z}\vec{a}_y$

$$\Rightarrow \beta = \omega\sqrt{\mu_0\varepsilon_0} = 10^8/3.10^8 = 1/3$$

$$\Rightarrow \dot{\vec{H}} = -\frac{1}{2\pi}.e^{-jz/3}\vec{a}_x$$

$$\Rightarrow \vec{H}(z,t) = -\frac{1}{2\pi}.\cos(10^8 t - z/3)\vec{a}_x \ (A/m)$$