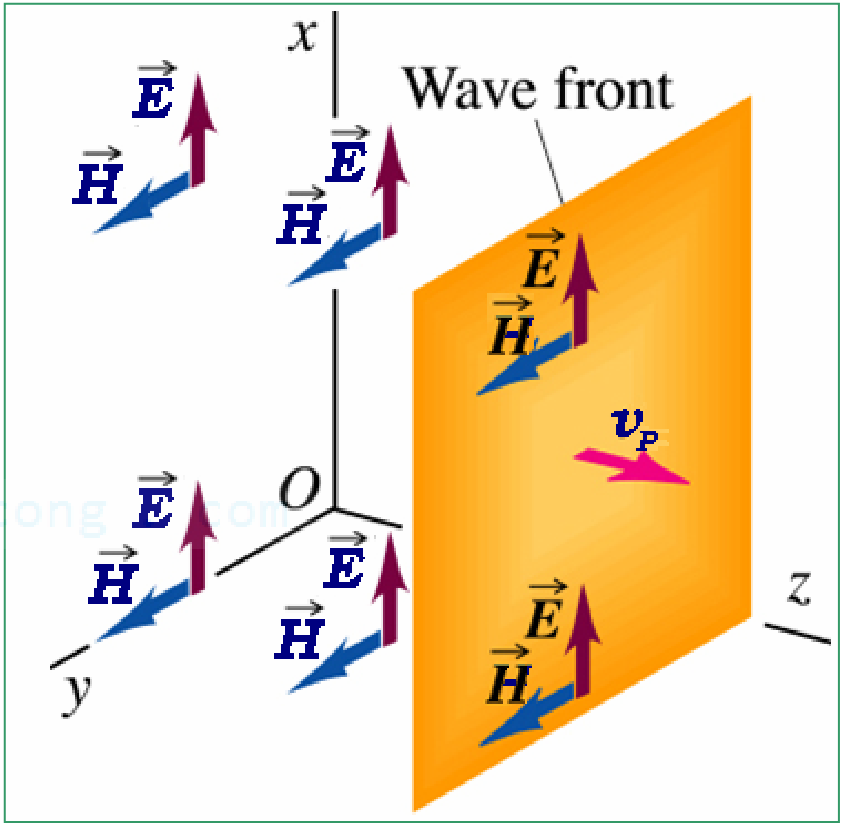
4.3 Uniform Plane Wave :

a) Introduction to uniform plane wave :

i. $\vec{E}$ and $\vec{H}$ lie in a plane perpendicular to the direction of propagation of wave.

ii. The field quantities have no components in the direction of propagation of wave. Called Transverse ElectroMagnetic wave (TEM wave).

iii. The field quantities have the same magnitude and direction in a plane containing it . → *uniform*



b) Uniform plane wave Equations :

- Assume the medium to be linear, homogeneous , isotropic and source-free ($\rho_v = 0$) .

Wave propagating in the z direction. The fields are sinusoidal and not functions of x and y.

$$\vec{E} = E_x \vec{a}_x = E(z)\cos[\omega t + \psi_E(z)]\vec{a}_x$$

$$\vec{H} = H_y \vec{a}_y = H(z)\cos[\omega t + \psi_H(z)]\vec{a}_y$$

The phasors :

$$\dot{\vec{E}} = E(z)\angle\psi_E(z)\vec{a}_x = \dot{E}.\vec{a}_x$$

$$\dot{\vec{H}} = H(z)\angle\psi_H(z)\vec{a}_y = \dot{H}.\vec{a}_y$$

b) Uniform plane wave Equations :

$$\text{rot}\,\dot{\vec{H}} = (\sigma + j\omega\varepsilon)\dot{\vec{E}}$$

$$\text{rot}\,\dot{\vec{E}} = -j\omega\mu\,\dot{\vec{H}}$$

$$\Rightarrow \Delta\dot{\vec{E}} - j\omega\mu(\sigma + j\omega\varepsilon)\dot{\vec{E}} = 0$$

And $\dot{\vec{E}} = \dot{E}(z).\vec{a}_x$ $\Rightarrow$

$$\frac{\partial^2 \dot{E}}{\partial z^2} - j\omega\mu(\sigma + j\omega\varepsilon)\dot{E} = 0$$

$\Rightarrow$

$$\dot{E} = M_1 e^{-\gamma z} + M_2 e^{\gamma z}$$

$$\dot{H} = \frac{M_1}{\eta} e^{-\gamma z} - \frac{M_2}{\eta} e^{\gamma z}$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta$$

(propagaton constant [m^{-1}])

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| \angle \tau$$

(intrinsic impedance[Ω])

c) Summary: u.p.w

i. **u.p.wave = incident + reflected**

$$\dot{E} = M_1 e^{-\gamma z} + M_2 e^{\gamma z}$$

$$\dot{H} = \frac{M_1}{\eta} e^{-\gamma z} - \frac{M_2}{\eta} e^{\gamma z}$$

ii. The phasors of incident wave :

❖ **None of the reflection :**

$$\dot{\vec{E}} = \dot{E}.\vec{a}_x = M_1 e^{-\gamma z}.\vec{a}_x = \dot{\vec{E}}_0 e^{-\gamma z}$$

$$\dot{\vec{H}} = \dot{H}.\vec{a}_y = \frac{M_1}{\eta} e^{-\gamma z}.\vec{a}_y = \dot{\vec{H}}_0 e^{-\gamma z}$$

$M_1 = m_1 \angle \varphi_1$ = magnitude/phase of electric field at z = 0.

$\dot{\vec{E}}_0, \dot{\vec{H}}_0$ = the phasors at z = 0.

iii. The propagation constant :

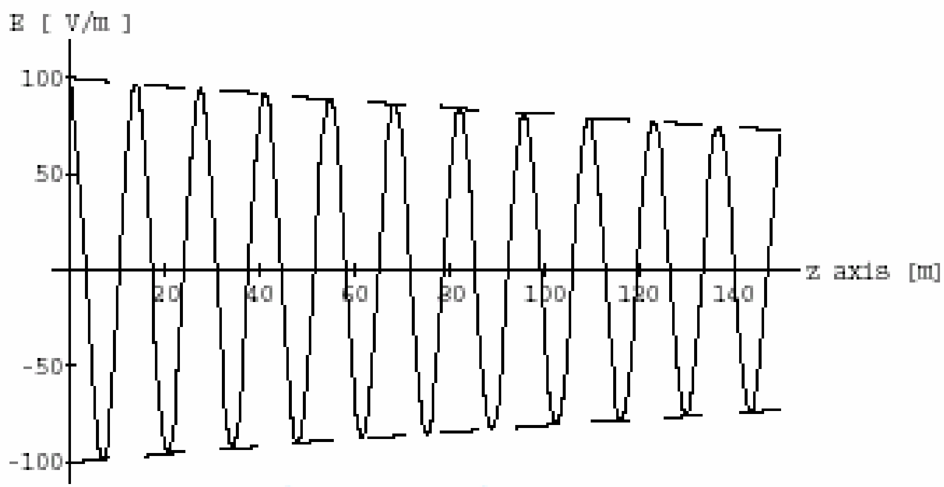
$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta$$

$$\dot{\vec{E}} = \dot{\vec{E}}_0 e^{-\gamma z} = [m_1 e^{-\alpha z} \angle(\varphi_1 - \beta z)]\vec{a}_x$$

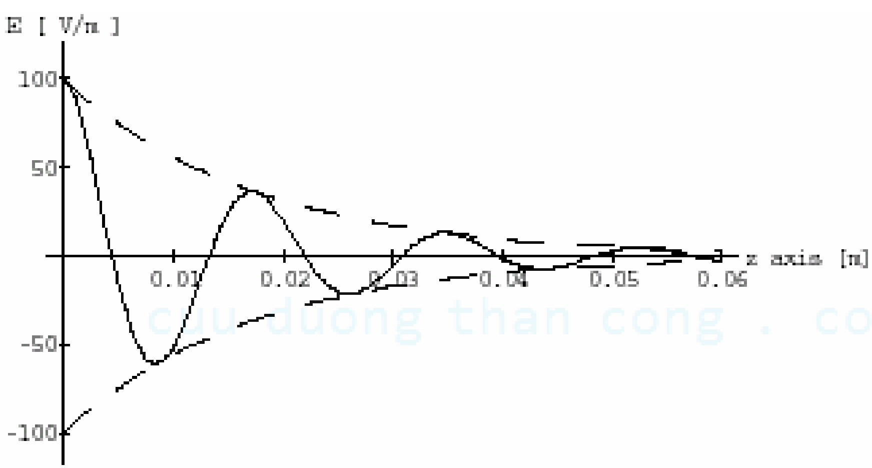
$$\alpha = \text{attenuation const} = \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{2}} \sqrt{\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} \quad \text{(Np/m)}$$

$$\beta = \text{phase const} = \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{2}} \sqrt{\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} \quad \text{(rad/m)}$$

Ví dụ mô tả sự suy giảm bđộ Tđiện :



Electric field intensity in distilled water ($\mu_r = 1$, $\varepsilon_r' = 80$, $\sigma = 2\times10^{-4}$ S/m.)



Electric field intensity in a piece of steak ($\mu_r = 1$, $\varepsilon_r' = 47$, $\sigma = 2.17$ S/m.)

iv. The intrinsic impedance :

- Là tỉ số biên độ phức trường điện / trường từ .

$$\eta = \frac{\dot{E}}{\dot{H}} = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| \angle \tau \quad (\Omega)$$

v. Relation between the electric and magnetic :

$$\dot{\vec{H}} = \frac{1}{\eta}\left[\vec{a}_S \times \dot{\vec{E}}\right] \qquad \dot{\vec{E}} = \eta\left[\dot{\vec{H}} \times \vec{a}_S\right]$$

$\vec{a}_s$ = propagating unit vector

- Example: if $\dot{\vec{E}} = [m_1 e^{-\alpha z}\angle(\varphi_1 - \beta z)]\vec{a}_x$

  And $\vec{a}_S = \vec{a}_z$; $\eta = |\eta|\angle\tau$

$$\Rightarrow \dot{\vec{H}} = \frac{m_1 e^{-\alpha z}}{|\eta|}\angle(\varphi_1 - \beta z - \tau)\vec{a}_y$$

- **In time-domain:**

$$\vec{E}(z,t) = m_1 e^{-\alpha z}\cos(\omega t - \beta z + \varphi_1)\vec{a}_x$$

$$\vec{H}(z,t) = \frac{m_1}{|\eta|}e^{-\alpha z}\cos(\omega t - \beta z + \varphi_1 - \tau)\vec{a}_y$$

vi. Wave front :

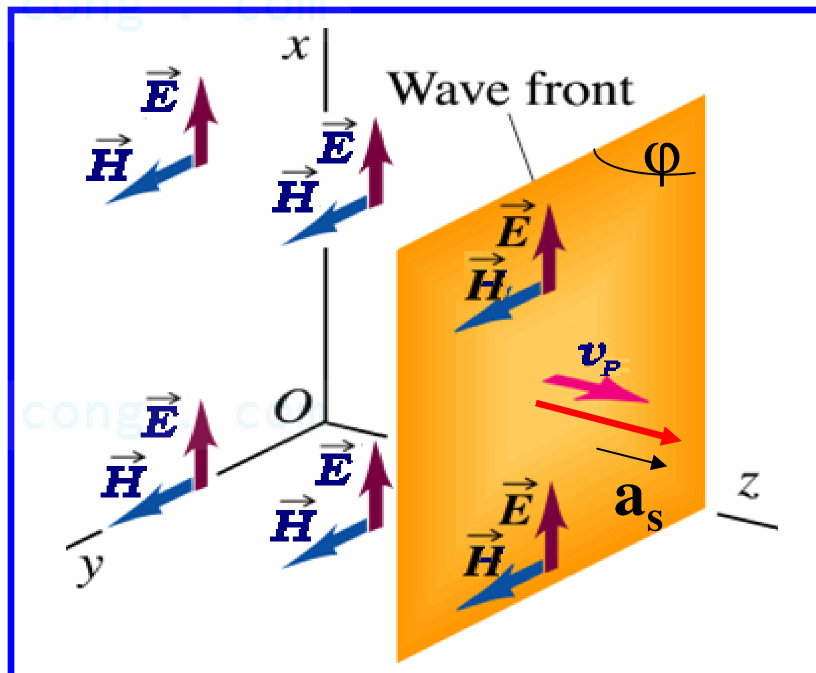
$$\text{the phase of wave} = (\omega t - \beta z + \varphi_1)$$

$$\text{wavefront: } (\omega t - \beta z + \varphi_1) = const \quad ; t = \text{const}$$

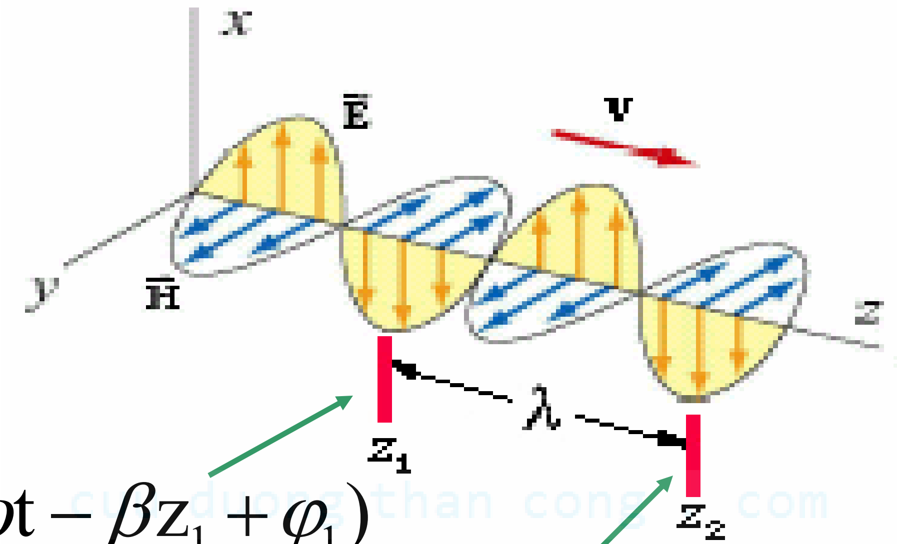
$\Rightarrow$ $z = const$ **Wave front perpendicular to Oz**

- **Velocity of the wavefront is given by :**

$$v_p = \frac{\omega}{\beta}$$



vii. Wavelength:



$$(\omega t - \beta z_1 + \varphi_1)$$

$$(\omega t - \beta z_2 + \varphi_1)$$

- **Distance between so that** $(\omega t - \beta z_1 + \varphi_1) - (\omega t - \beta z_2 + \varphi_1) = 2\pi$

$$\Rightarrow \quad \lambda = (z_2 - z_1) = \frac{2\pi}{\beta}$$

viii. The loss tangent:

$$d = \text{tg}\theta = \text{the loss tangent} = \frac{\sigma}{\omega\varepsilon}$$

❖ **Quy ước :**

$$\text{tg}\theta = \begin{cases} < 10^{-1} : \text{lossy dielectric} . \\ > 10^{1} : \text{good conductor} . \\ 10^{-1} \div 10^{1} : \text{general medium} . \end{cases}$$

ix. The complex permittivity :

Assume that: $j\omega\tilde{\varepsilon} = \sigma + j\omega\varepsilon$

$$\tilde{\varepsilon} = \varepsilon - j\sigma/\omega = \varepsilon_0(\varepsilon_r - j\sigma/\omega\varepsilon_0)$$

$$\gamma = j\omega\sqrt{\mu\tilde{\varepsilon}}$$

$$\eta = \sqrt{\frac{\mu}{\tilde{\varepsilon}}}$$

❖ Độ thẩm điện phức của vật liệu :

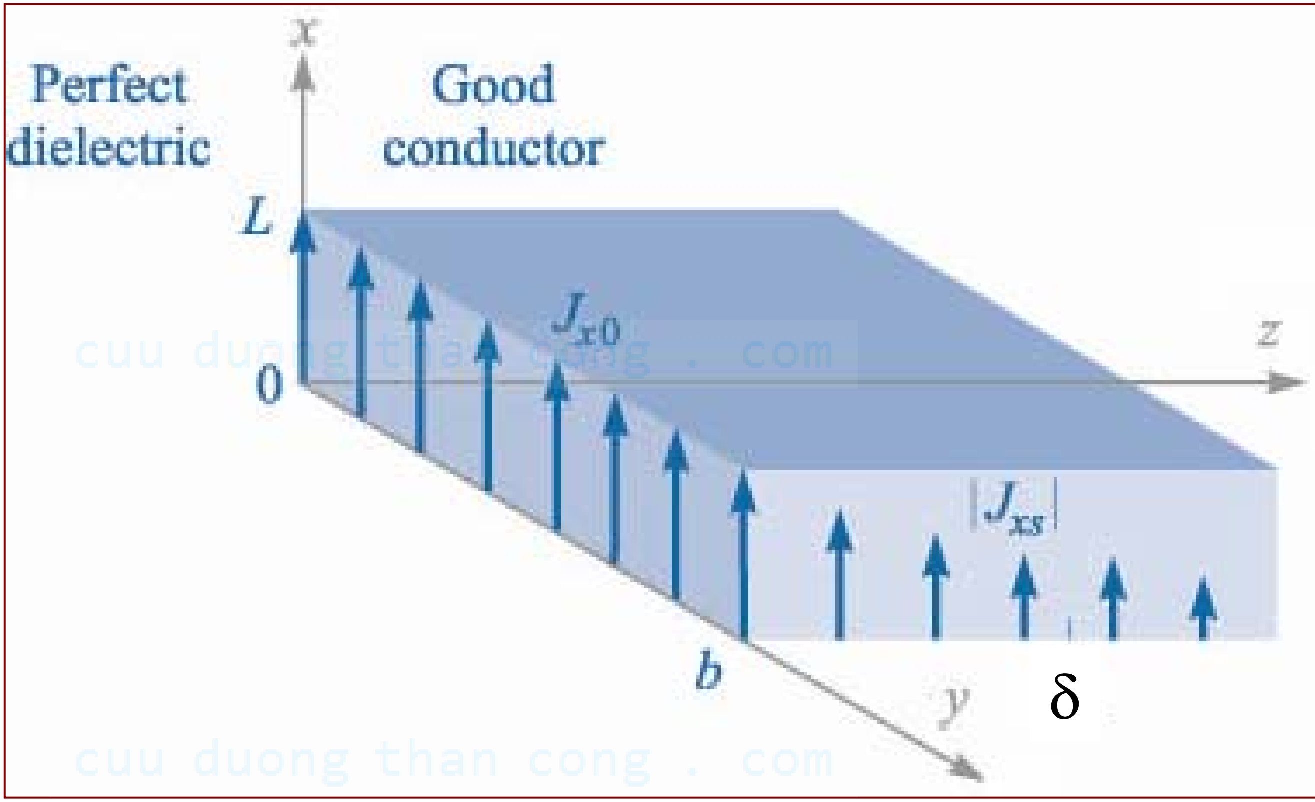
$\varepsilon = \varepsilon_o(\varepsilon_r - j\varepsilon'')$ where $\varepsilon'' = \gamma/\omega\varepsilon_0$ and $\varepsilon_o \approx 10^{-9}/36\pi$

Material*	ε_r	γ (mho/m)	ε'' at 1 GHz
Lime stone wall	7.5	0.03	0.54
Dry marble	8.8		0.22
Brick wall	4	0.02	0.36
Cement	4 - 6		0.3
Concrete wall	6.5	0.08	1.2
Clear glass	4 - 6		0.005 - 0.1
Metalized glass	5.0	2.5	45
Lake water	81	0.013	0.23
Sea Water	81	3.3	59
Dry soil	2.5	--	--
Earth	7 - 30	0.001 - 0.03	0.02 - 0.54

x. The skin depth :

❖ The distance δ : field attenuated by the factor $e^{-1} = 0.368$.

$$\delta = \frac{1}{\alpha}$$



❖ At the distance of 5δ : field is vanished .