4.4 Poynting Theorem :

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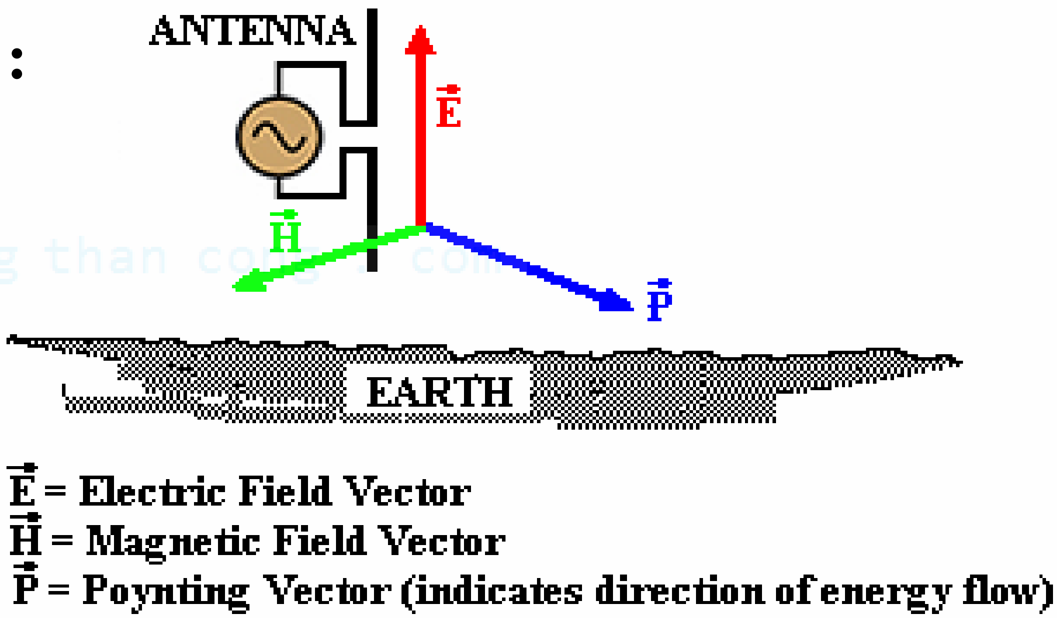
a) Instantaneous Poynting vector $\vec{P}$:

- Defined by : $$\vec{P} = \vec{E} \times \vec{H} \quad (W/m^2)$$

- Power flow pass the area S :

$$P_S = \int_S \vec{E} \times \vec{H} \, d\vec{S}$$



- Power flow into the closed surface S :

$$P_S = -\oint_S \vec{E} \times \vec{H} \, d\vec{S}$$

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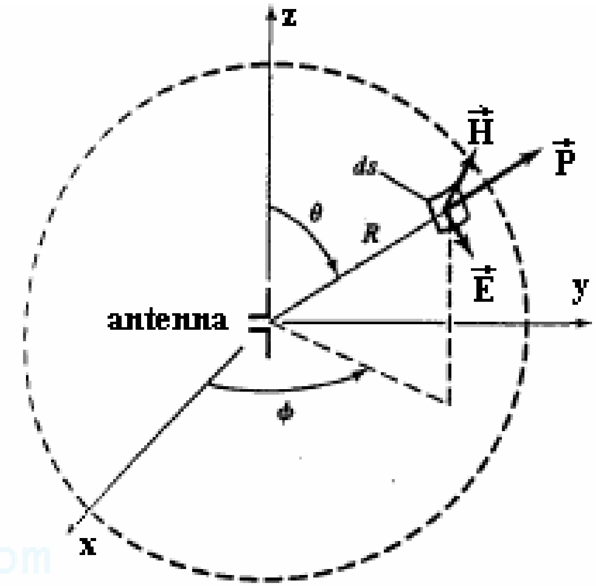
b) Poynting's theorem :

$$P_S = -\oint_S \vec{E} \times \vec{H}\, d\vec{S}$$



$$\ldots \quad P_S = \int_V \vec{E}\,\vec{J}\,dV + \int_V \left(\vec{E}\frac{\partial \vec{D}}{\partial t} + \vec{H}\frac{\partial \vec{B}}{\partial t}\right) dV$$

$$\ldots \quad P_S = \int_V \vec{E}\,\vec{J}\,dV + \frac{d}{dt}\int_V \frac{1}{2}\left(\vec{E}\,\vec{D} + \vec{H}\,\vec{B}\right) dV$$

$$\Rightarrow \quad P_S = P_J + \frac{dW}{dt}$$ *(Poynting's theorem)*

P_J = EM Energy dissipated as heat in V

$W = \frac{1}{2}\int_V (\vec{E}\vec{D} + \vec{H}\vec{B})dV = W_e + W_m$ = EM Energy stored in V

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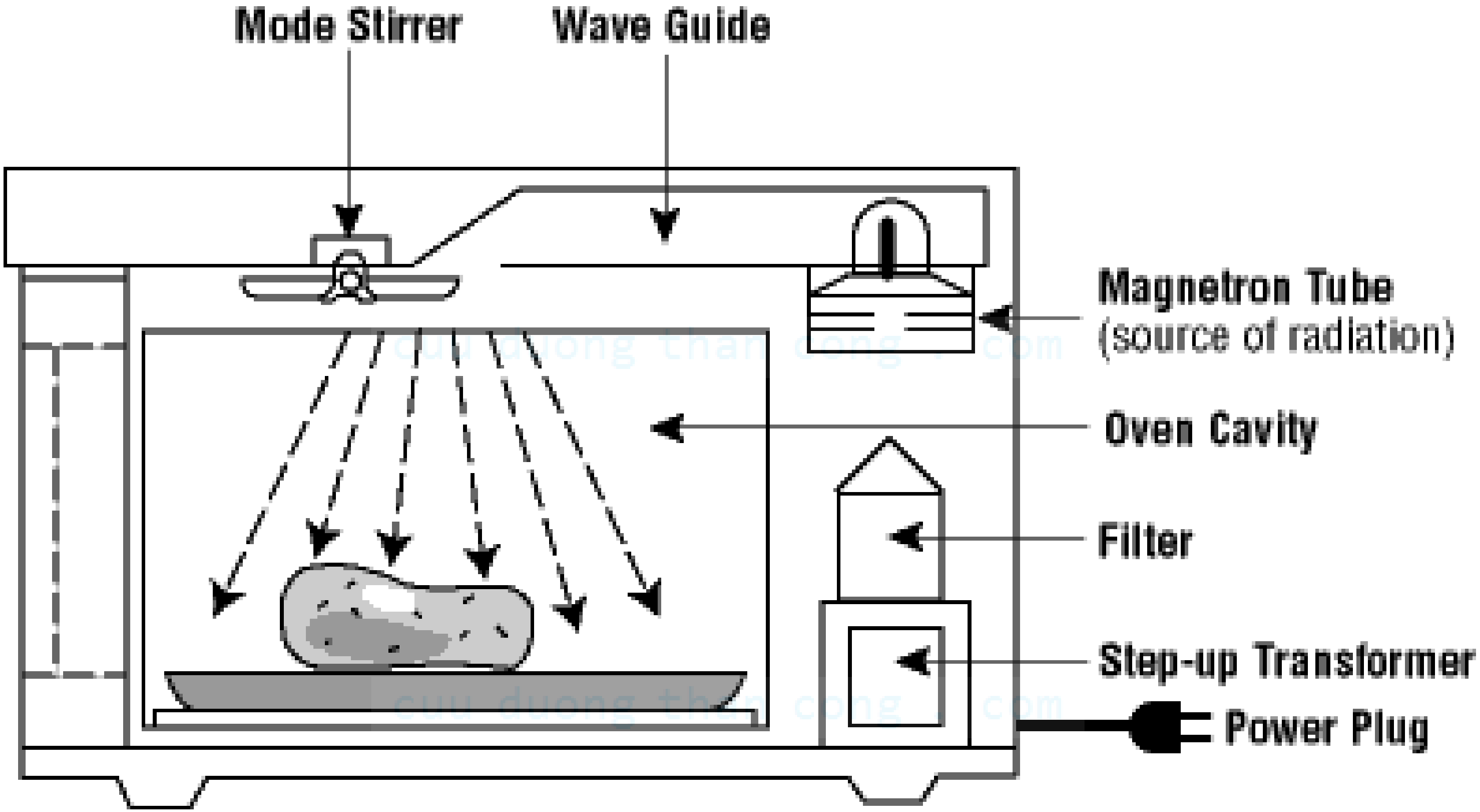
Application: Microwave oven



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c) Sinusoidal electromagnetic wave:

- **Complex Poynting vector :** $$\tilde{\vec{P}} = \frac{1}{2}\left\{\dot{\vec{E}} \times \overset{*}{\vec{H}}\right\}$$
- **Time-average Poynting vector :** $$\left\langle \vec{P} \right\rangle = \mathrm{Re}\left\{\tilde{\vec{P}}\right\}$$
- **Time-average power density :** $$\left\langle P_s \right\rangle = \left\langle \vec{P} \right\rangle . \vec{a}_s$$

$$\Rightarrow \quad < P_s > = \frac{1}{2} E_m^2 \,\mathrm{Re}\left[\frac{1}{\eta}\right] \qquad < P_s > = \frac{1}{2} H_m^2 \,\mathrm{Re}\left[\eta\right]$$

- **Time-average dissipated power density :** $$< p_J > = \frac{1}{2} \sigma E_m^2$$

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