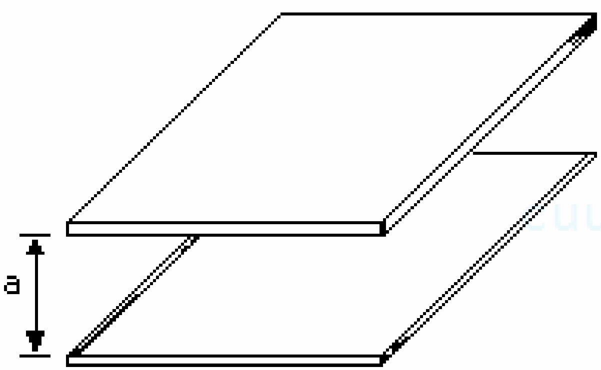
# Chapter 6: Waveguide

# 6.1 Introduction :

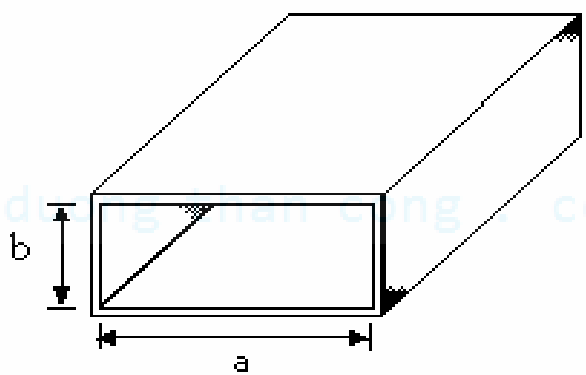
- **Transmision line:**
  - Two conductors
  - $f < f_{cut}$
  - TEM wave

- **Waveguide:**
  - Only one hollow conductors
  - $f > f_{cut}$
  - TE and TM waves

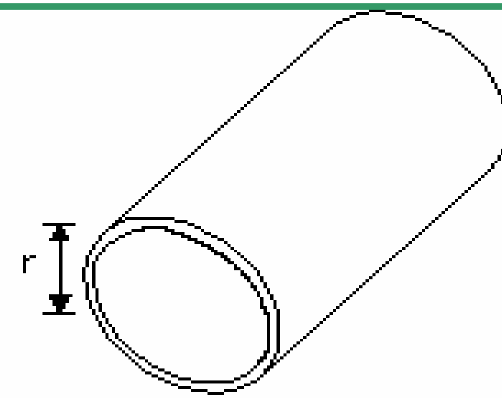





a) Parallel-plate waveguide



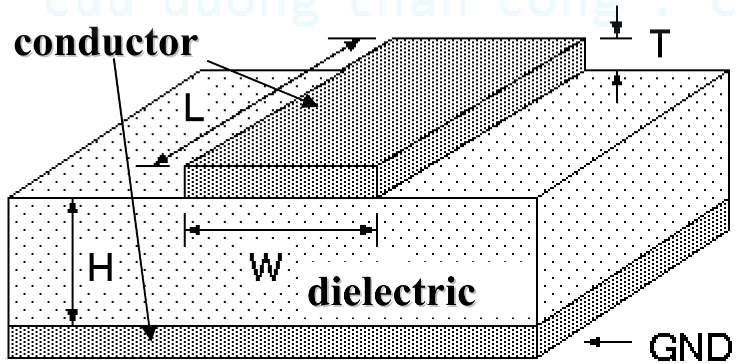

b) Rectangular waveguide



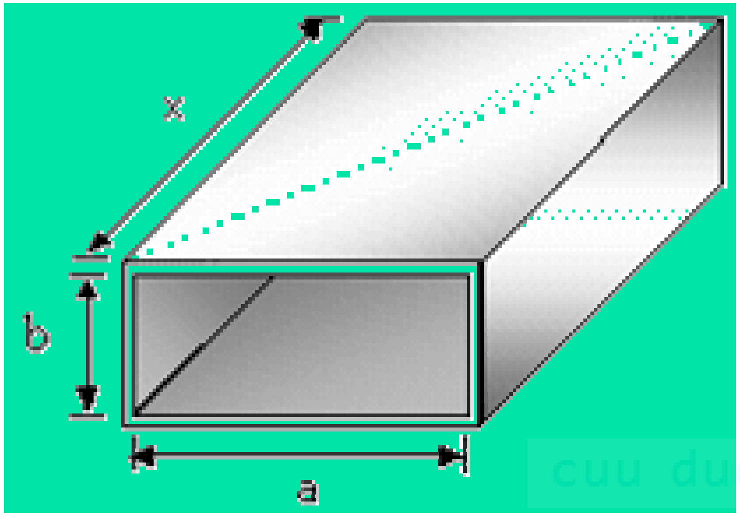

c) Circular waveguide

# ❖ Microstrip line :



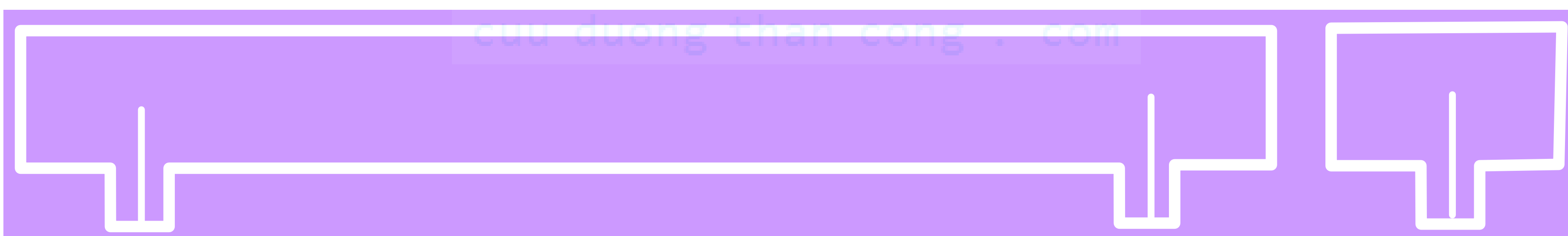



# ❖ Rectangular Waveguide:

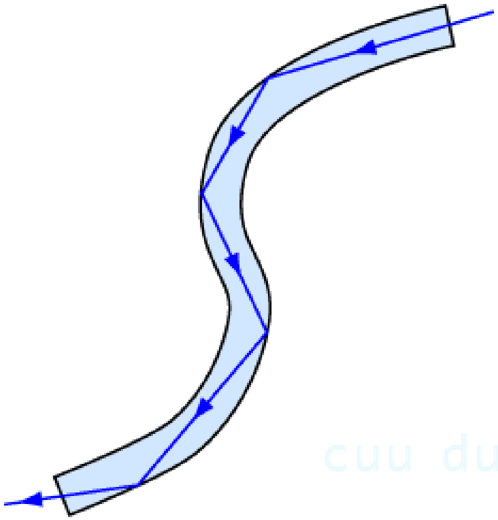


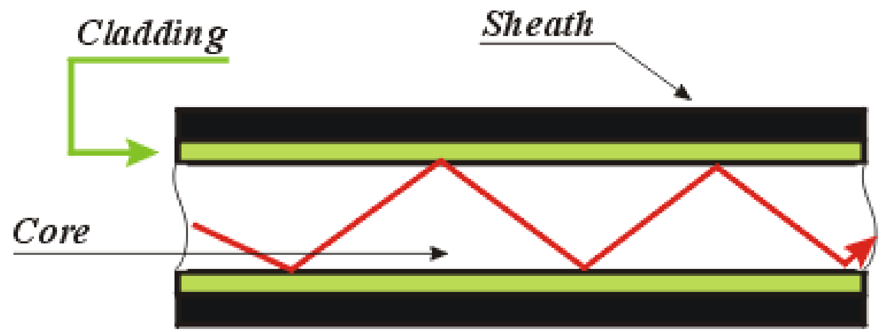

rectangular waveguides

cross section

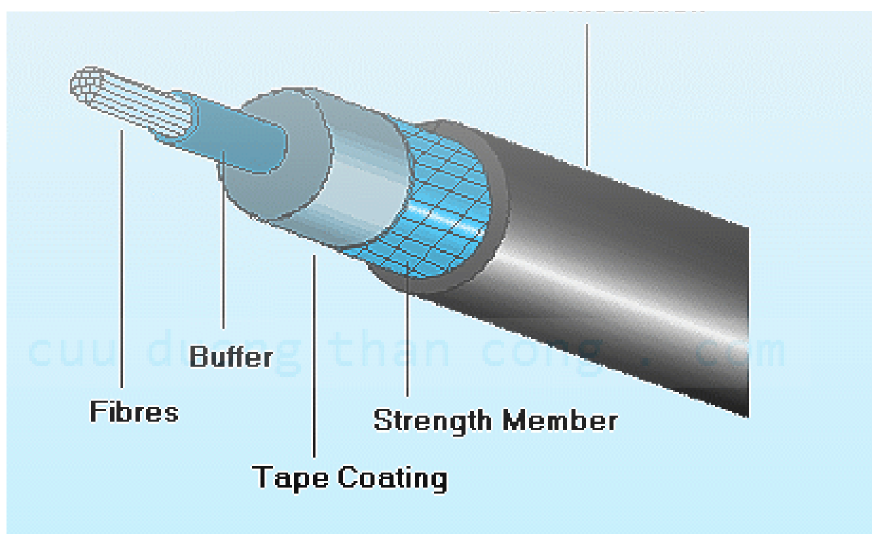


# ❖ Optical Fibre:

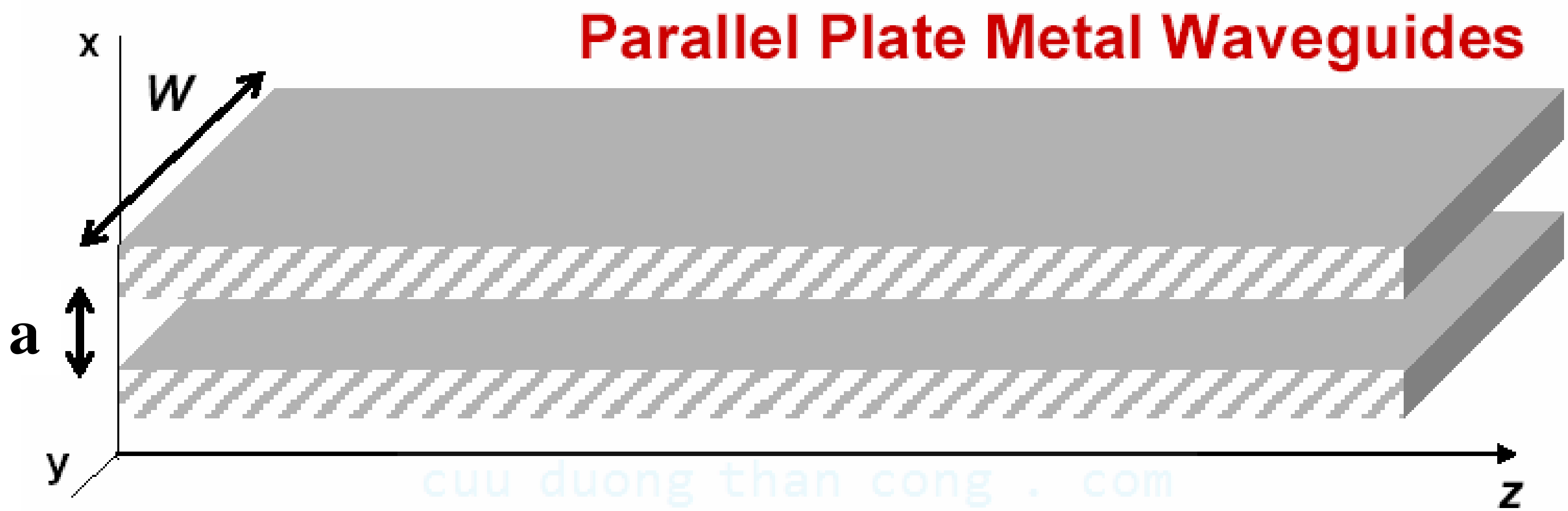






# 6.2: Parallel-plate waveguide:



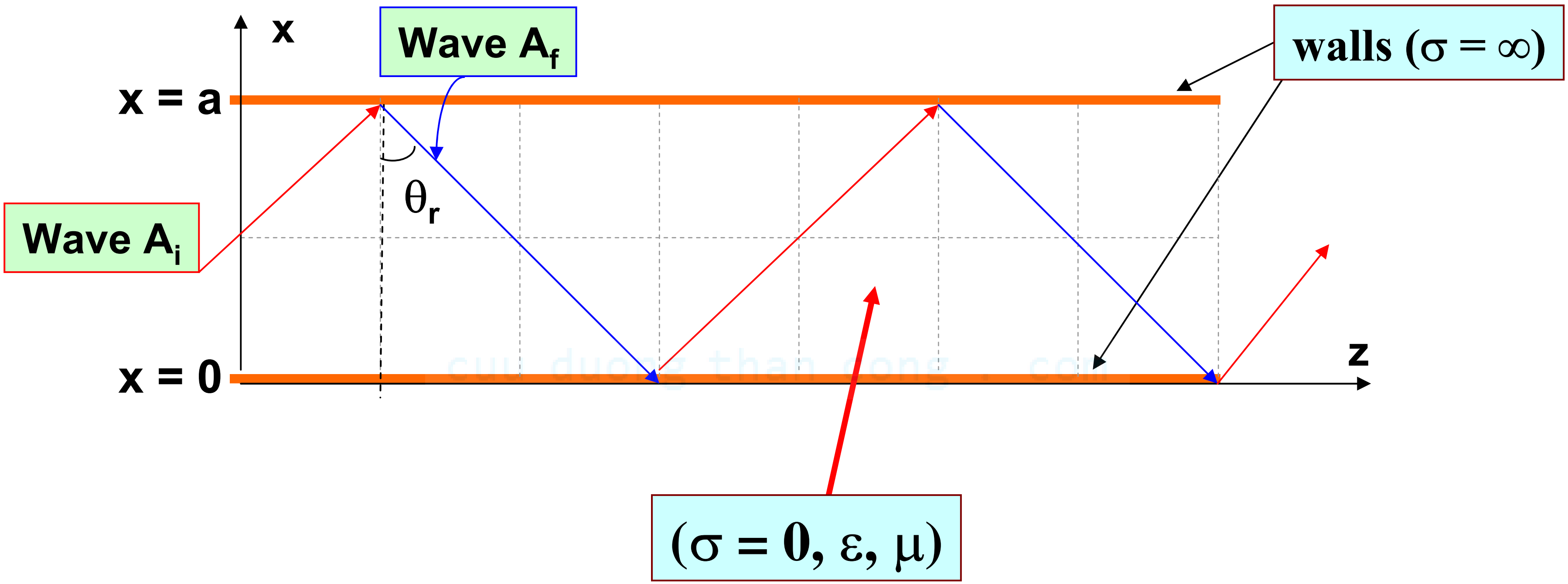



- Consider a parallel plate waveguide (shown above)
- We have studied such structures in the context of transmission lines
- We know that they can guide TEM waves (Transverse Electric and Magnetic) in which both the electric and magnetic fields point in direction perpendicular to the propagation direction

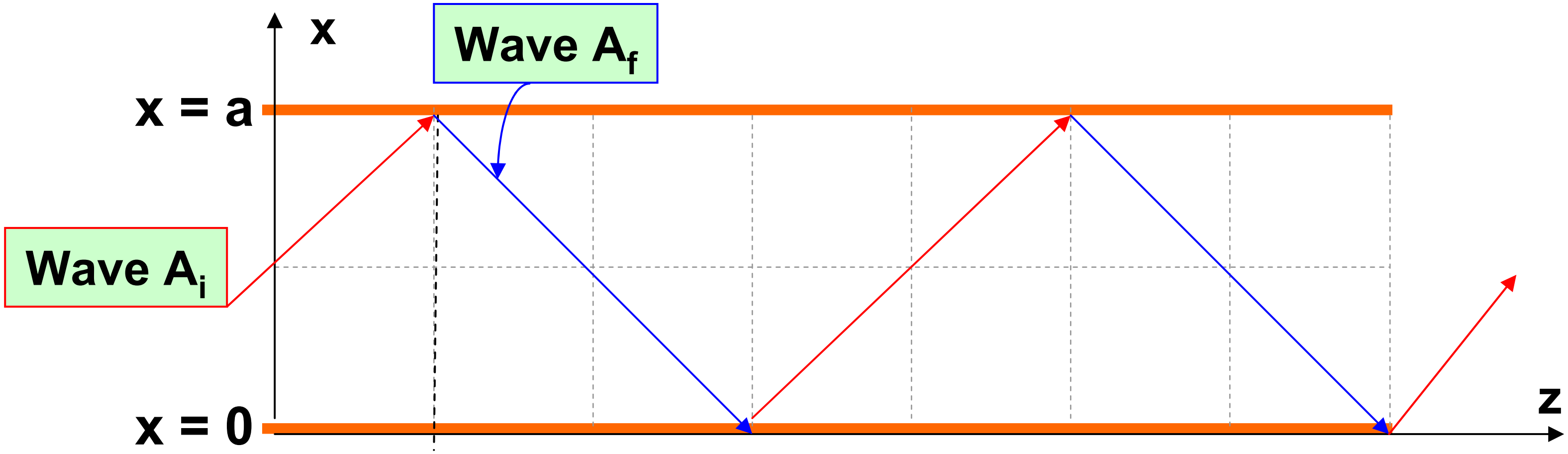


- But these structures can guide more than just the TEM waves that we have considered so far .............

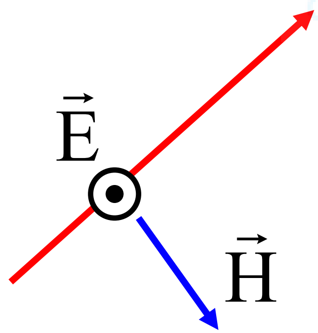
# a) The Principle:

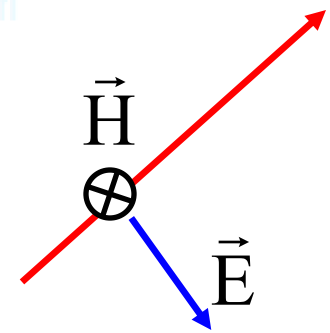


# b) The Guide Modes:






$$\vec{E} = E_y \vec{a}_y$$

$$\vec{H} = H_x \vec{a}_x + H_z \vec{a}_z$$

⇨ **TE wave**



$$\vec{E} = E_x \vec{a}_x + E_z \vec{a}_z$$

$$\vec{H} = H_y \vec{a}_y$$

⇨ **TM wave**

## c) The TM Guidewave Modes :

- Assume sinusoidal field. In phasor form :

$$\dot{E}_x = E_x(x).e^{-\gamma z} \;\; ; \;\; \dot{E}_z = E_z(x).e^{-\gamma z}$$

$$\dot{H}_y = H_y(x).e^{-\gamma z}$$

- Maxwell's equations :

$$\text{rot}\,\vec{\dot{H}} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & 0 & \partial/\partial z \\ 0 & \dot{H}_y & 0 \end{vmatrix} = j\omega\varepsilon\,\vec{\dot{E}} \Rightarrow \begin{cases} \gamma\dot{H}_y = j\omega\varepsilon\dot{E}_x & (1) \\ \dfrac{\partial\dot{H}_y}{\partial x} = j\omega\varepsilon\dot{E}_z & (2) \end{cases}$$

$$\text{rot}\,\vec{\dot{E}} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & 0 & \partial/\partial z \\ \dot{E}_x & 0 & \dot{E}_z \end{vmatrix} = -j\omega\mu\vec{\dot{H}} \Rightarrow -\gamma\dot{E}_x - \frac{\partial\dot{E}_z}{\partial x} = -j\omega\mu\dot{H}_y \quad (3)$$

## ❖ The component $\dot{E}_z$ :

$$\dot{E}_x = \frac{\gamma}{j\omega\varepsilon}\dot{H}_y \Rightarrow \frac{\partial \dot{E}_z}{\partial x} + \left(\frac{\gamma^2}{j\omega\varepsilon} - j\omega\mu\right)\dot{H}_y = 0$$

$$\Rightarrow \frac{\partial^2 \dot{E}_z}{\partial x^2} + (\gamma^2 + \omega^2\mu\varepsilon)\dot{E}_z = 0 \qquad (\gamma = j\beta_Z)$$

$$\Rightarrow \boxed{\frac{\partial^2 \dot{E}_z}{\partial x^2} + K_c^2 \dot{E}_z = 0}$$

$$(K_C^2 = \omega^2\mu\varepsilon + \gamma^2 = \omega^2\mu\varepsilon - \beta_Z^2)$$

## ❖ The other components :

$$-\frac{\partial \dot{E}_z}{\partial x} = \left(\gamma + \frac{\omega^2 \mu\varepsilon}{\gamma}\right)\dot{E}_x \quad \Rightarrow \quad \dot{E}_x = -\frac{\gamma}{K_C^2}\frac{\partial \dot{E}_z}{\partial x}$$



$$\dot{E}_x = \frac{\gamma}{j\omega\varepsilon}\dot{H}_y \quad \Rightarrow \quad \dot{H}_y = \frac{\dot{E}_x}{\eta_{TM}} \quad \left(\eta_{TM} = \frac{\beta_Z}{\omega\varepsilon}\right)$$

## The $TM_m$ wave :

$$\dot{E}_Z = C\sin\left(\frac{m\pi}{a}x\right).e^{-\gamma Z}$$

**m = 0, 1, 2, 3 ...**
***( mode index)***

***(Note: m = 0 : TEM wave)***

$$\dot{E}_x = -\frac{\gamma}{K_C^2}\frac{\partial \dot{E}_z}{\partial x}$$

$$\dot{H}_y = \frac{\dot{E}_x}{\eta_{TM}}$$

$$K_C^2 = \left(\frac{m\pi}{a}\right)^2$$

$$\gamma = j\beta_Z$$

$$\beta_Z = \sqrt{\left(\frac{\omega}{v}\right)^2 - \left(\frac{m\pi}{a}\right)^2}$$

$$(\eta_{TM} = \frac{\beta_Z}{\omega\varepsilon})$$

# d) The $TE_m$ wave :

$$\frac{\partial^2 \dot{H}_z}{\partial x^2} + K_c^2 \dot{H}_z = 0$$

$$\dot{H}_Z = C\cos\left(\frac{m\pi}{a}x\right).e^{-\gamma Z}$$

**m = 1, 2, 3 ...**
***( mode index)***

***(Note: m = 0 : $H_x$ & $E_y$ = 0)***

$$\dot{H}_x = -\frac{\gamma}{K_C^2}\frac{\partial \dot{H}_z}{\partial x}$$

$$\dot{E}_y = -\eta_{TE}.\dot{H}_x$$

$$(\eta_{TE} = \frac{\omega\mu}{\beta_Z})$$

## e) Summary :

**i. Cutoff-frequency:**

$$f > \frac{v}{2}.\frac{m}{a} = f_c \qquad \lambda = \frac{v}{f} < \frac{2a}{m} = \lambda_c$$

**ii. Phase constant:**

$$\beta_Z = \sqrt{\left(\frac{\omega}{v}\right)^2 - \left(\frac{m\pi}{a}\right)^2} = \beta\sqrt{1-(f_C/f)^2}$$



❖ **If f < f$_C$ :** $\gamma^2 = (m\pi/a)^2 - \omega^2\mu\varepsilon > 0$

$$\Rightarrow \alpha_Z = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \left(\frac{\omega}{v}\right)^2} \Rightarrow$$ **Attenuation !!!**

# iii. Velocity :

$$v_m = \frac{\omega}{\beta_z} = \frac{v}{\sqrt{1-\left(\frac{f_C}{f}\right)^2}} \geq v$$

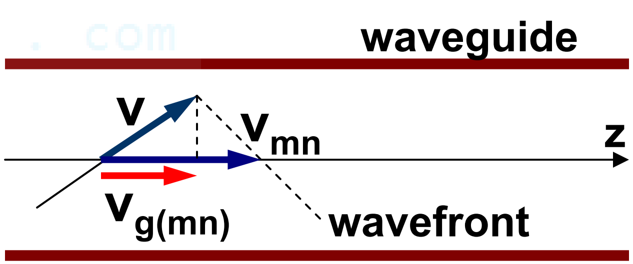
❖ ***Nhận xét:***

a) Vận tốc pha trong ods luôn lớn hơn vận tốc truyền sóng trong không gian tự do .

b) Khi truyền tín hiệu , TĐT biến thiên sẽ bị điều chế để mang theo thông tin. Lúc này , vận tốc của nhóm sóng (vận tốc của các mặt phẳng vuông góc trục z) là cần thiết :

Và người ta chứng minh được :

$$v_{g(mn)} . v_{mn} = v^2$$



**iv. Wavelength:**

$$\lambda_m = \frac{2\pi}{\beta_Z} = \frac{\lambda}{\sqrt{1-(f_C/f)^2}}$$

**v. Wave impedance:**

$$\eta_{TE} = \frac{\eta}{\sqrt{1-(f_C/f)^2}}$$

$$\eta_{TM} = \eta\sqrt{1-(f_C/f)^2}$$